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DESCRIPTION OF A PLANNED DEMONSTRATION OF MASS DETERMINATION BY AN OSCILLATING MASS/SPRING SYSTEM

1.0 After theoretical study of the proposed and possible weightless mass measurement techniques, it appears that an oscillating mechanical system in which period varies as a function of mass is the most promising. The practicality and accuracy of various measuring techniques, accuracy desired, limits imposed by the characteristics of the human body to be measured and the constraints imposed by the conditions of measurement have all been considered in this conclusion which was shared by a number of people contacted who have theoretical and/or practical experience in the field including Mr. McNish, Chief Metrology Division NBS; Dr. Everett Palmatier, Chairman Department of Physics, UNC; Mr. Frank Riley, Project Engineer of Lockheed Aircraft and Dr. Ritter, Physicist, Aerospace Medical Division. A second unanimous opinion is that little further is to be gained from theoretical studies without an accompanying practical experiment(s).

The problems in demonstrating the accuracy of such a system may be arbitrarily divided into three areas:

(1) theoretical

(2) instrumental details for earth demonstration

(3) flight system

1.1 Theoretical:

The solutions of the general differential equations of an oscillating system have been solved in detail since the same equations describe a number of physical systems including mechanical, electrical and acoustic.

The idealized shown in Figure 1 system consists of a mass M, attached to a restoring force $F_1=KX$, provided by a massless spring attached to a rigid support. Motion of the particle is assumed to create a linear resisting force of $R\frac{dx}{dt}$.

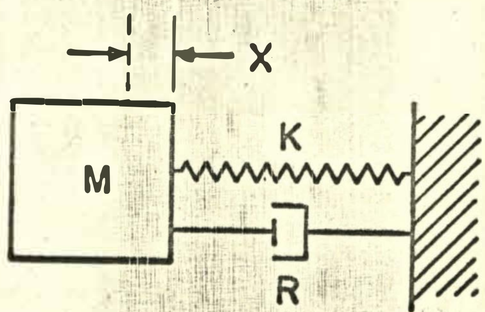

Fig. 1.1 Mechanical System Moving in X axis with Mass-M, Spring-K, and Resistance-R.

The general equation of motion of such a system undergoing "natural" oscillation in a single plane, i.e., displaced from its position of rest and allowed to return with outside influence is:

$$M\frac{d^2x}{dt^2} + R\frac{dx}{dt} + KX = 0 \qquad \text{Eq. 1.1}$$

There are two possible forms to the solution of the equation depending upon the relative amount of resistance present. If the resistance, R, is equal or greater than $2\sqrt{KM}$ then the mass will return to the position of equilibrium in an exponentional fashion. If, as the case will be here, $\frac{R}{2\sqrt{KM}} < 1$, an oscillation about the equilibrium point will result. If the resistance is zero, an undamped or continuous oscillation will occur whose frequency is given by:

$$F_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{KG}{W}} \qquad \text{Eq. 1.2}$$

Where: F_n = undamped natural frequency (cycles per second) with R=0

K = spring constant $\frac{(\text{pounds})}{(\text{inches})}$

M = mass $\frac{(\text{pounds}\cdot\text{second}^2)}{(\text{inches})} = \frac{W\ (\text{pounds})}{G\ (\text{inches/second}^2)}$

W = weight (pounds)

G (Austin, Texas) $979.283\ \frac{\text{cm}}{\text{Sec}^2}$

$= 385.54372\ \frac{\text{inches}}{\text{seconds}^2}$

1 inch = 2.540005 cm.

The undamped natural period T_n = time (seconds) = $\frac{1}{F_n}$

$$T_n = \frac{1}{F_n} = 2\pi\sqrt{\frac{W}{KG}} \qquad \text{Eq. 1.3}$$

Amplitude will be equal to the amplitude of the original displacement as in Fig. 1.2A.

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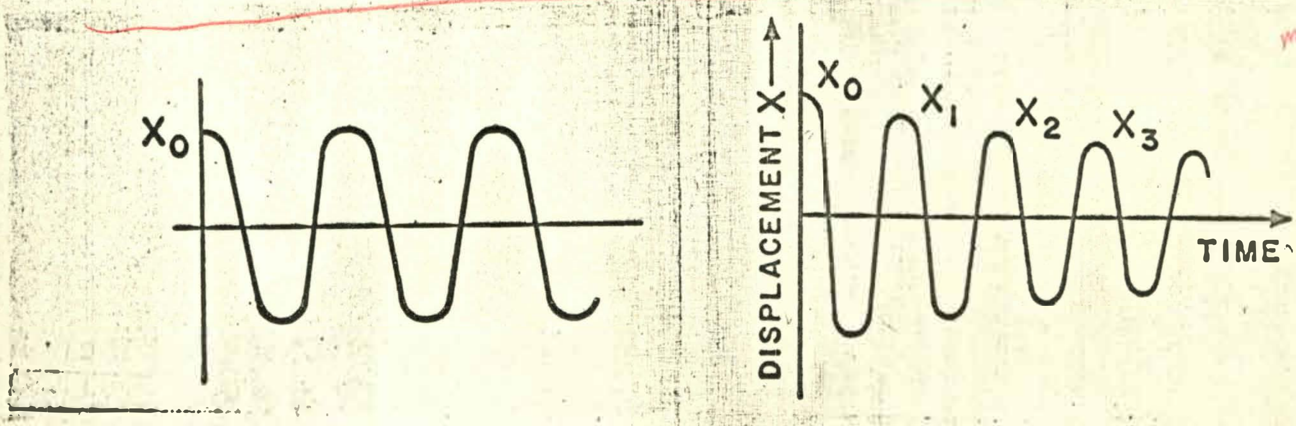


Fig. 1.2 A. Undamped Natural Oscillation R=0 B. Damped Natural Oscillation $0 > R < 2\sqrt{KM}$

If the resistance is not zero, i.e., energy is dissipated as in any practical system, both the frequency and amplitude will be modified

$$\cancel{\frac{\Delta M}{M} = \frac{2\Delta T}{T}}$$

from the undamped case as illustrated in Fig. 1.2B.

The new frequency will be given by:

$$F_d = F_n \sqrt{1 - \left(\frac{R}{2\sqrt{KM}}\right)^2} \qquad \text{Eq. 1.4}$$

F_d = damped natural frequency where $R \neq 0$ (cycles per second). In addition, the peak amplitude will decrease by

$$\frac{Xn}{Xo} = e^{-\frac{2\pi n K_d}{\sqrt{1 - K_d^2}}} \qquad Kd = \frac{R}{2\sqrt{KM}} \cancel{= \sqrt{1 - \left(\frac{T_n}{T_d}\right)^2}}$$

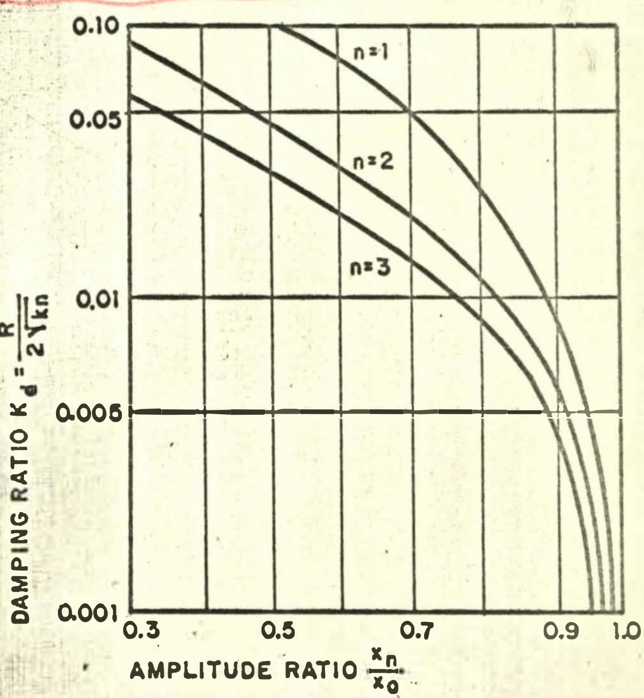
Xo = amplitude of original displacement

Xn = peak amplitude of successive damped oscillations

N = number of cycles,

= 1, 2, 3 - - - - -

Fig. 1.3 Amplitude of the First Three Cycles of Damped Oscillation Plotted as a Function of Damping Ratio



The two fundamental questions that arise from purely theoretical considerations then are concerned with period (frequency) determinations and knowledge of R, K&M to determine deviations from the natural frequency. The minimum time resolution ΔT required for idealized measurement of a mass change of .1 pound is calculated here for a typical case of a 150 pound object at a period of approximately 1.2 seconds.

T_1 - period (seconds) at 150 pounds

T_2 - period (seconds) at 150.1 pounds

$\Delta T = T_2 - T_1$

$W_1 = 150$ pounds

$W_2 = 150.1$ pounds

$K = 10$ pounds/inches

$$T = 2\pi\sqrt{\frac{W}{KG}} \qquad \text{Eq. 1.6}$$

$G = 385.5437$ inches/second^2

$T_2 = 1.2397444$ seconds

$T_1 = 1.2393297$ seconds

$\Delta T = .4147 \times 10^{-3}$ seconds $\sim$ .5 m Sec

This order of time resolution may be readily obtained by a counter of 10^{-4} seconds resolution (10^{-6} seconds resolution is routine) so measurement of time per se is not difficult, but it is obvious that a stop watch will not suffice. As will be shown later, the difficulty will arise from the distance resolution required to obtain this time.

The next consideration, effects of resistance, cannot be determined with such accuracy. The resistance in the real case will not be lumped but will consist of several components which do not readily lend themselves to precise calculation. The two chief sources of resistance arise from: 1) the motion of the mass (man) through a viscous medium (air), 2) the dissipation of the spring. While the spring resistance (hysteresis) will remain a small system constant, the viscous resistance (air drag) will vary with subject configuration and surrounding atmosphere and cause variable deviations from the undamped natural frequency. Since the body will be irregular and variable and the atmosphere is presently unknown, only an estimation may be made of this effect.

Air "drag" at appreciable velocities may be calculated from:

$$D = C_d SP \qquad \text{Eq. 1.7}$$

P = dynamic pressure = $\frac{V^2 \rho}{2}$ (slugs second^2/feet)

D = drag (pounds)

C_d = Coefficient of drag

= 1.28 for flat plate

S = Area (feet^2)

ρ = density = 2.38×10^{-3} slugs/feet^3 for normal atmosphere

V = velocity $\frac{\text{(feet)}}{\text{(second)}}$

Resistance $R = \frac{\text{Force}}{\text{Velocity}}$ and combining this with the drag equation:

$$R = \frac{CdSPV^2}{2\ V} = \frac{CdSPV}{2} \qquad \text{Eq. 1.8}$$

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Assuming a flat plate area of 3 feet^2 for a seated man in a normal atmosphere with a velocity of 1 foot/second (this is greater than our maximum velocity will be), the resistance is calculated to be

$$R = \frac{1.23 \times 3 \times 2.38 \times 10^{-3} \times 1}{2} = 4.57 \times 10^{-3} \text{ pounds}\cdot\text{second/feet}$$

The Aeromed lab uses values of CdS of 5 feet^2 minimum to 10 feet^2 maximum for the clothed human body. Taking the 10 feet^2 value will result in a R of $\sim 1.5 \times 10^{-2} \frac{\text{pounds}\cdot\text{second}}{\text{feet}}$

We may next note the maximum allowable effects of resistance for the accuracies desired (auxillary calibration would be possible but undesirable).

We have seen that an accuracy of $\sim .5\times10^{-3}$ seconds in time is required for a change of .1 pound of 150 pounds and for convenience these time figures will be used here. Eq. 1.3 allows calculation of the effect of R and rearranging this in terms of period Eq. 1.4 will allow one to calculate the maximum tolerable R for the given ΔT assuming there are no spring losses or other system errors.

$$K_d = \sqrt{1-\left(\frac{T_n}{T_d}\right)^2} \qquad \text{Eq. 1.9}$$

R = Mechanical resistance $\left(\frac{\text{pounds}\cdot\text{second}}{\text{inches}}\right)$

K = Spring constant = 10 $\frac{(\text{pounds})}{(\text{inch})}$

$M = \frac{W}{g}$ = Mass = $\frac{150 \text{ pounds}}{g}$

$= .389 \frac{(\text{pounds}\cdot\text{second}^2)}{(\text{inch})}$

T_n = Undamped period (seconds)

= 1.2393297

T_d = damped period (seconds) = 0.2397444

$$K_d = \sqrt{1-\left(\frac{1.2393297}{1.2397444}\right)^2} = \sqrt{1-.99933} = 2.58\times10^{-2}$$

Also: $K_d = \frac{R}{2\sqrt{KM}}$

$$R = 2\,K_d\sqrt{KM}$$

$$= 2 \times 2.585 \times 10^{-2} \times 10 \times .389$$

$$= 10.18 \times 10^{-2}\ \frac{\text{pounds} \cdot \text{second}}{\text{inch}}$$

This figure is almost on order of magnitude greater than our worst case total drag. Variations in this drag figure should have negligible effect on the mass determination.

Another consideration is the limited mass of the space ship. This limited mass results in a small amplitude of oscillation of the ship which will shift the frequency of oscillation as follows:

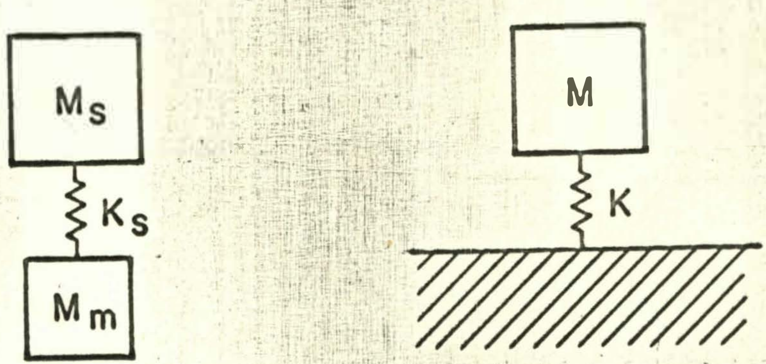

Fig. 1.4 A. Spring/Mass System Attached to Finite Mass B. Spring/Mass System Attached to Infinite Mass

M = Mass to be measured

Ms = Mass of ship

K_s = Spring of Constant

$$F = \frac{1}{2\pi}\sqrt{\frac{K_s(Mm+Ms)}{MmMs}}$$

This has the effect of an apparently smaller measured mass, M_E and is equal to $\frac{MmMs}{Mm+Ms}$

Using values of:

$$Mm = \frac{150}{g} \text{ pounds}$$

$$Ms = \frac{30,000}{g} \text{ pounds}$$

$$Me = \frac{3x10^4 x1.5x10^2}{(3.+ .0150)x10^4} = \frac{4.5 \times 10^6}{3.015x10^4}$$

$$= 149.254 \text{ pounds}$$

In absolute terms the weight would be "short" by approximately .75 pounds. This error will remain essentially constant without large variations in the ship's mass and may be calculated or calibrated by a single on-station measurement with a known mass.

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2.0 There seem to be no fundamental theoretical difficulties so a realistic demonstration of the technique under earth conditions should be made.

It must be pointed out that the Lockheed demonstration of this method was crude, apparently for lack of materiel support. Unfortunately, the idea is extant that the results represented what might be reasonably expected from such a system. Mr. Riley, the project engineer, is well aware of this and feels that the required accuracies of $\pm$.1% may be obtained in orbit and/or demonstrated on earth with adequate instrumentation.

The essential components of an experiment on earth to demonstrate the validity of theoretical conclusions consist of:

2.1 structure to hold the mass to be weighed

2.2 a spring assembly

2.3 a device to determine precisely when the oscillating mass crosses point of zero displacement

2.4 a counter for determining time periods

2.5 some means of providing displacement along a given axis

2.6 on earth, some method of supporting the weight without adding appreciable friction.

This latter item makes the device more difficult to demonstrate under gravity than under weightlessness.

The experiment will be described by these sections. The basic arrangement is shown in Figure 2.1.

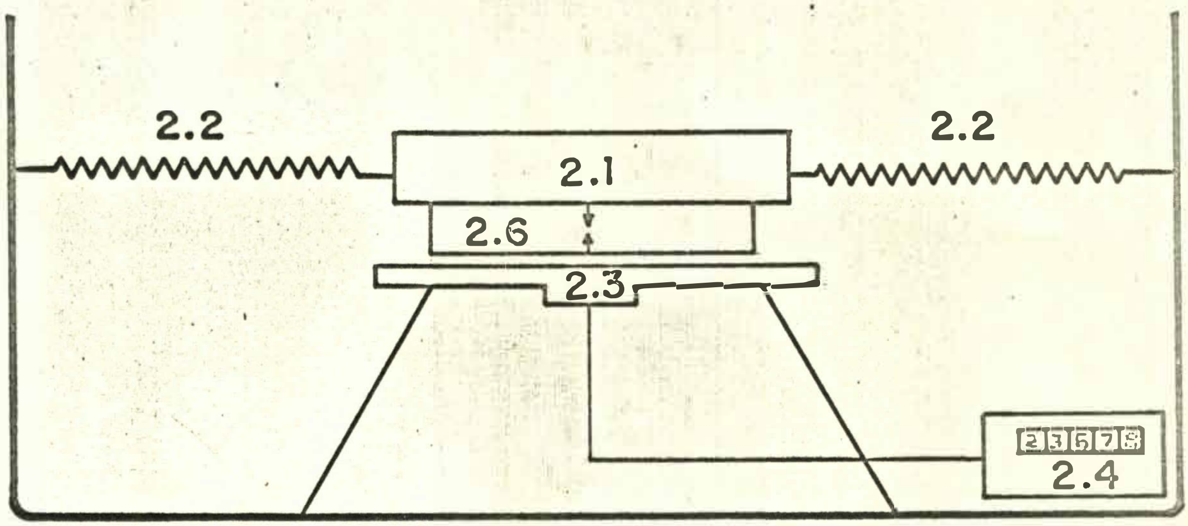


Fig. 2.1 Arrangement of System for Measurement of Mass. Parts Legend is on P-10

2.1 The structure to hold the mass is complicated by the nature of the human body and indeed the whole experiment will be more or less influenced by these considerations.

The human body may be considered to consist of a complex series of subunits. A 1^{o} order approximation is shown in mechanical analogs in Figure 2.2.

These segments constitute a series of mechanical systems capable of interdependent oscillation. If, however, the frequency of oscillation is low enough, the viscous damping forces become sufficiently large to allow the system to be treated as a single mass.

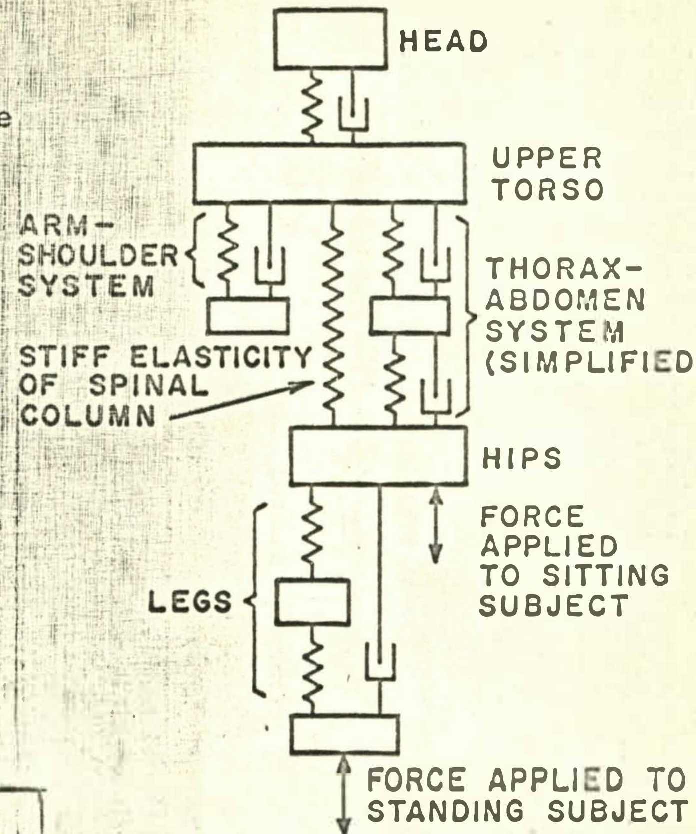


Fig. 2.2 Second Order Mechanical Analog of Human Body After Coermann

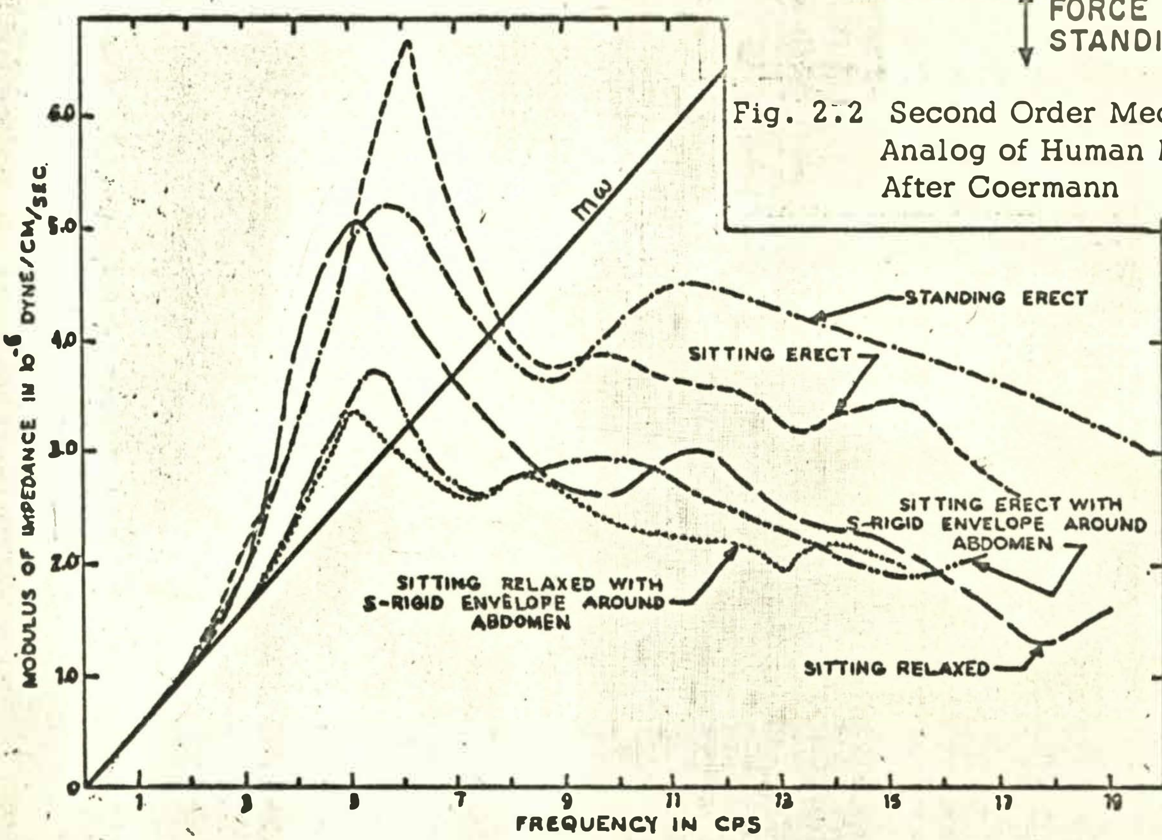


Fig. 2.3 Mechanical Impedance of the Human Body Under Several Conditions - Coermann, et al.

A plot of impedance of the body as a function of frequency of vibration is shown in Fig. 2.3. The impedance of a fixed mass is shown by the straight line. Any deviation from this line would result in an error in the proposed system. From these and a wide variety of similar experiments by Coermann, it appears that the maximum frequency which should be considered is less than one cycle per second.

A second aspect of the human body is that it cannot remain fixed in one position for any great length of time which 1) limits the time available for measurement and 2) makes center of mass measurements of such variability as to be of academic interest only. Any mass measurement scheme should be arranged such that these changes in mass relationships (movement) will cause the least possible disturbance. One approach to this is to limit the degrees of freedom of motion to the minimum, which will be simple translatory motion along a single axis. Even so, errors will be introduced by movement, as anyone knows who has tried to weight a child on an ordinary scale. This error will be present here only during the cycle(s) when movement occurred. By allowing the major muscles of the body to contract against a mechanism rigidly attached to the spring system three benefits should accrue: 1) subject motion should be reduced to a minimum, 2) the body will be more rigidly attached to the spring system and 3) the compliance of the body's elasticity will be greatly reduced, i.e., the body will more nearly approximate a fixed mass.

Movement may be further reduced by breath holding. The ballistic effects of the cardiovascular system will remain. This latter effect will be small and will be essentially zero over a complete cardiac cycle. For a human mass measurement system then, the "weighing pan" should have a foot board and hand holds or other arrangement to allow contraction of the major musculature. The limited amount of time that a fixed position may be held is a limitation of the maximum period of time such a measurement may be made. This measurement period should be as long as possible for as will be shown later, accuracy should vary as the square root of the time duration of measurement. A practical limit would seem to be 10-15 seconds.

2.2 Spring Assembly. Several factors are of importance here including the energy dissipative characteristics (hysteresis), linearity and time, dimensional and temperature stability. When one speaks of springs we are referring to the precision devices which are in the majority of commercial scales in the country today rather than the crude devices used to close doors and the like.

Stability. The time stability of precision springs may be neglected for our purposes.

A spring under load will elongate slowly, an effect called "creep". This effect is shown in 2.4. A spring is subjected to a

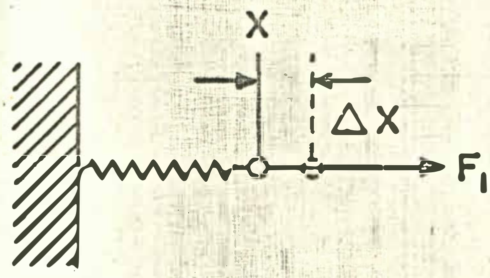


Fig.2.4 Creep or Elongation of Spring Under Load, F_1 - X is Original Position

fixed force F_1 and depending upon its constant K, it will be stretched to a position X_1 from the spring equation F=KX. Depending upon the time and temperature, the spring will gradually elongate to a new position $X_1 + \Delta X$.

Since the force is unchanged, this is equivalent to a new spring constant $K_2 = \frac{F}{X + \Delta X}$. The creep of a good spring is typically .01%/24 hours under maximum load which results in a change of .01% in spring constant. Push pull springs will be used here which will double this effect to .02%. However, by simply unloading the spring assembly except when in actual use this effect may be neglected. This normally occurs in spring scales which are unloaded when not in use.

The temperature coefficient of the planned spring material may be varied during manufacture from 0 to 15×10^{-6}. The temperature variations of the lab should also be small.

After conversation with individuals with extensive experience with springs, there appears to be no exact expression available for the hysteresis of a spring so that calculations of resistance would only be estimates and will not be done here. This factor should be negligible as the hysteresis will be .02% on the units to be used*. A number of other effects such as exceeding elastic limits might be present in a poor design but will not be present here. A minor effect could result from twisting with displacement but this will be avoided by compound winding.

To summarize, the springs should not present a problem if the best material available is used.

2.3 The accuracy with which the point of zero crossing is determined is fundamental to the accuracy of the entire system. Although the basic measurement is time, it is dependent upon the distance resolution of the zero crossing detector and velocity of the mass at zero crossing as follows:

$$\text{Time Resolution} = \frac{\text{Distance Resolution}}{\text{Velocity at zero displacement}}$$

Taking a nominal frequency of 1 cycle per second with a peak displacement of 1 inch will give a peak velocity of

$$\frac{dxo}{dt} = 2\pi \ FXo \sim 6.28 \ \frac{\text{inches}}{\text{seconds}}$$

For a time resolution of 5 milliseconds the distance resolution must then be:

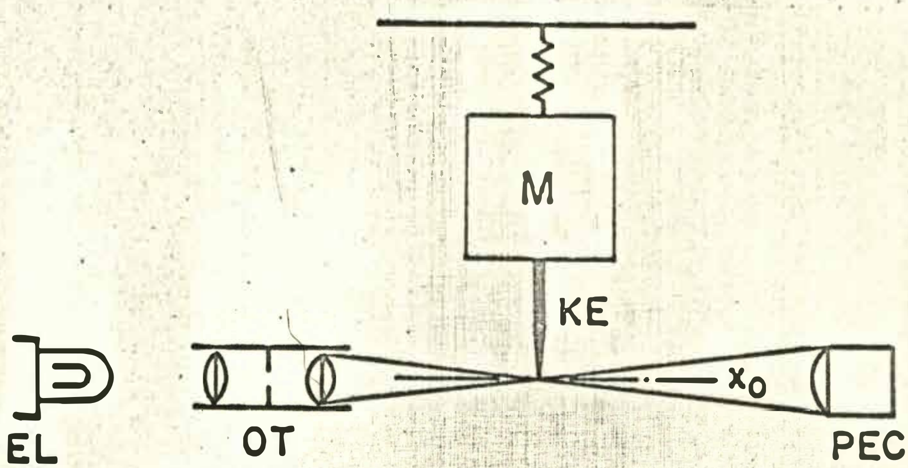
*Some measurements recently made here on the resistance losses (hysteresis) show that these losses are very negligible.

Distance = Velocity x Time

$$= 6 \frac{\text{inches}}{\text{seconds}} \times .5 \times 10^{-3} \text{ seconds}$$

$$= 3. \times 10^{-3} \text{ inches}$$

A narrow beam of light interrupted by a knife edge has been chosen for the zero crossing detection method. This is shown diagrammatically in Figure 2.5.



An incandescent bulb E.L. illuminates a slit in the optic tube assembly which is then focussed as a vertical ribbon of light .001 inch wide x ~ .1 inch deep at Xo. A knife edge K.E. is attached to the oscillating mass M and moves across the axis X. At all points above Xo the photo electric cell PEC is illuminated fully and provides a maximum light output. At Xo the light is cut off in .001 inch and remains off when the mass is below this point. The system thus has an inherent accuracy of .001 inch.

This is further enhanced by the electronic circuitry shown in Figure 2.6 as follows: The voltage e_1 is the output of the photocell while e_2 is fixed at 1/2 the maximum voltage of e_1.

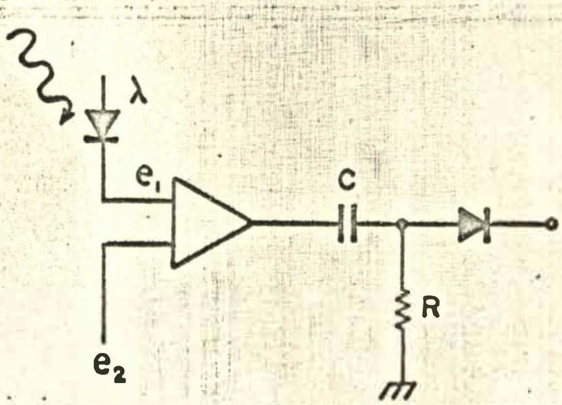


Fig 2.6 Simplified zero crossing detector circuit.

A is a stable high gain differential amplifier that will provide an output of approximately 20 volts for a 1mV. difference between e_1 and e_2.

The slope of the PEC output is several volts/10^{-3} inch as the knife crosses the light beam. This results in a theoretical resolution of microinches. Practically there is some motion in other planes with defocussing and other effects. A measured series of curves with controlled errors introduced have been made and show that a resolution of 10^{-4} inches may be reasonably expected. This is more than an order of magnitude greater than the required resolution.

2.4 The output of the zero crossing detector must then be converted to signals controlling the timer. A simple R.C. differentiator will generate a pulse each time zero is crossed. Since zero is crossed twice each cycle and from alternately opposite directions, only pulses of a single polarity will be passed to the counter which will then count every other complete cycle.

This arrangement will be used only in the testing phases for investigative purposes. The arrangement which will be used during mass measurement will consist of timing a fixed number of cycles. Within the limits imposed the greater the number of cycles counted the greater the accuracy, assuming random errors. The time accumulated will be a direct function of the number of counts while the random errors will accumulate as the square root of the number of counts. The signal to error ratio will then increase as the square root of the number of cycles taken. For example, if random errors existed in the time measurement such that the individual cycle resolution were only 1 millisecond, by counting over 4 cycles this could be reduced to the required 1/2 milli-second resolution. The timer used in the experiment will be an ordinary Hewlett Packard with a maximum time resolution of 10^{-6} seconds. Overall system time resolution should be $\sim 5 \times 10^{-5}$ seconds.

2.5 A system for initial displacement should present no great problem. The magnitude of initial displacement will be a compromise among several factors including distance resolution available, working distance of springs, space available and bearing size. A nominal distance of 1 inch total peak to peak displacement has been chosen here. Since the bearing used will allow only rectilinear motion some form of simple stop at $\sim$ .5 inch maximum deflection with a positive manual release is needed.

2.6 Essentially frictionless support is limited to a few choices here of which the most obvious is an air bearing. A quick and dirty approximation of such a bearing is a block of dry ice supporting a smooth surface. Such makeshift is not justified here and a simple linear air bearing with lateral and bottom surfaces will be used. Such a bearing may for our purposes be considered frictionless. An air supply of a few P.S.I. and low volume will be required for operation.

The first phase of the demonstration will consist of assembling the components and testing the validity of the derived calculations including the resolution, accuracy and stability of the proposed instrument using fixed masses. A 2° phase will consist of construction of a man carrying version and development and test of the features peculiar to this application. If successful, a third phase should be considered which consists of flying the two devices with cylindrical bearings to determine their behavior under weightlessness and to gain operational experience with this principal. Information and guidance should be provided to the contractor during all phases as soon as valid results are obtained.

3.0 Assuming that the principles described will hold, one must then determine the possibility of achieving a practical system under conditions imposed by space flight. These conditions include severe limitations on size, weight, power, complexity (particularly of operation) as well as a variety of environmental conditions including vibration and G loading during powered flight, sub-normal atmospheric pressure and possibly unusual atmospheric composition.

The oscillating mass scheme lends itself well to such conditions. The condition of weightlessness will be an asset. The only bearings required under zero G are to restrain small forces from deviating the mass out of the desired axis of oscillation. A small pair of cylindrical air bearings operating with 1-2 PSI are all that should be required. The air flow rate will also be low such that a small bleed from a bottle containing atmospheric gases or a small pump would be needed for the brief period of measurement.

The reduced atmospheric pressure with its lowered viscosity will also be an asset.

The system is relatively simple. In the case of mass determination of the crew, a modification of a normal crew position seat may be used as shown in Fig. 3.1. During lift off and normal periods, the movable portion of the seat would be clamped to the rigid structure contiguous with the ship. The spring assembly on the back would also be unloaded by a release arrangement. For mass determination the spring would be placed in tension and the seat unclamped and allowed to move upward a short distance. The crew member would grasp the side hold and exert pressure against the foot boards to provide body rigidity and firm attachment to the seat.

A small, probably less than one inch, displacement would be made and a few cycles of oscillation would take place along the axis of the twin air bearings restraining seat to ship. An optical pickoff would count the zero crossings and electronic circuitry would time this for display or recording.

The weight cost of this would be small, a few pounds maximum. Additional space required would be minimal. The zero crossing detector and counter are simple electronic devices and would likewise cause little penalty. An on-board precision oscillator for the tape recorder could also supply the timing frequency but this too is small and simple. The information may be in binary form for direct recording. The power requirement would be a few watts and then only for the period of oscillation.

In addition to the measurement of crew mass, there may be a number of quantitative procedures in which a small scale of high precision could provide a mass measurement much more easily than a volumetric measurement may be made. Such a mass scale is sketched in Fig. 3.2.

In short, this system with refinement should provide a method of determining mass under weightlessness with almost the same precision and ease that our conventional gravemetric scales now provide on earth.

William Thornton
Capt. M.C., U.S.A.F.
15 Oct., 1965