# BMMD CALIBRATION REPORT (BLIND STUDY) 

BY : PRADIP SHAH GE GOVT. SERVICES

The objective of this study is to obtain the calibration curve equation for the data collected from the Body Mass Measurement Device (BMMD). The numerical technique used to obtain the equation is the interpolation with the divided difference method.

## DIVIDED DIFFERENCE METHOD

The treatment of divided difference table assumes that a function, $f(x)$, is known at several distinct values for $x$ :

$$
\begin{array}{lll}
x_{0} & f_{0} \\
x_{1} & f_{1} \\
x_{2} & f_{2} \\
x_{3} & f_{3} \quad \text { and so on. }
\end{array}
$$

The x's are not assumed to be evenly spaced nor the values are arranged in any particular order (in the case of BMMD, the x's are readings and f's are masses). DEFINITION OF INTERPOLATING POLYNOMIALi Consider the nth degree polynomial:

$$
\begin{aligned}
P_{n}(x)=a_{0} & +\left(x-x_{0}\right) a_{1}+\left(x-x_{0}\right)\left(x-x_{1}\right) a_{2}+\ldots \ldots \ldots \ldots \ldots \ldots . . . . . . . . . . . . . . . . . . . . . . ~ \\
& +\left(x-x_{0}\right)\left(x-x_{1}\right) \ldots \ldots
\end{aligned}
$$

If we choose $a_{i}$ so that $P_{n}(x)$ equals $f(x)$ at the $n+1$ known points, $x_{0}, x_{1}, \ldots \ldots . ., x_{n}$, then $P_{n}(x)$ is an interpolating polynomial. Also, for the interpolating polynomial, it must match the table for all $n+1$ entries:

$$
P_{n}\left(x_{i}\right)=f_{i} \quad \text { for } i=0,1,2, \ldots \ldots \ldots . . . ., n .
$$

It can be shown that the above $a_{i}$ are readily determined by using what are called the divided differences of the tabulated values. A special notation is used for divided differences:

$$
f\left[x_{0}, x_{1}\right]=\left(f_{1}-f_{0}\right) /\left(x_{1}-x_{0}\right) \text { is called the first divided }
$$

difference between $x_{0}$ and $x_{1}$. The second and higher order divided differences are defined in terms of lower order difference. For example:

$$
\begin{aligned}
& f\left[x_{0}, x_{1}, x_{2}\right]=\left\{\left\{\left[x_{1}, x_{2}\right]-f\left[x_{0}-x_{1}\right]\right\} /\left(x_{2}-x_{0}\right) .\right. \\
& f\left[x_{0}, x_{1}, \ldots \ldots . x_{n}\right]=\left\{f\left[x_{1}, x_{2}, \ldots \ldots, x_{i}\right]-f\left[x_{0}, x_{1}, \ldots \ldots \ldots, x_{i-1}\right]\right\} /\left(x_{i}-x_{0}\right) .
\end{aligned}
$$

This concept is extended to a zero order difference : $\mathfrak{f}\left[\mathrm{x}_{\mathrm{s}}\right]=\mathrm{f}_{\mathrm{s}}$. Using this notation, a divided difference table, in symbolic form, is shown below.
$\left.\begin{array}{llllllll}x & f & {\left[x_{1}, x_{1}\right]} & f\left[x, x, \ldots, x_{1}\right] & f[x, x, x, x\end{array}\right]$

Using the definition of interpolating polynomial, the polynomial obtained will be,

$$
\begin{aligned}
& P_{3}(x)=f_{0}+\left(x-x_{0}\right) f\left[x_{0}, x_{1}\right]+\left(x-x_{0}\right)\left(x-x_{1}\right) f\left[x_{0}, x_{1}, x_{2}\right]+\ldots . . \\
& \ldots \ldots+\left(x-x_{0}\right)\left(x-x_{1}\right) \ldots \ldots \ldots . . .\left(x-x_{3}\right) f\left[x_{0}, x_{1}, x_{2}, x_{3}\right] .
\end{aligned}
$$

Similarly, for $n+1$ data points, the polynomial will be,

$$
\begin{aligned}
& P_{n}(x)=f_{0}+\left(x-x_{0}\right) f\left[x_{0}, x_{1}\right]+\left(x-x_{0}\right)\left(x-x_{1}\right)\left\{\left[x_{0}, x_{1}, x_{2}\right]+\right. \\
& +\left(x-x_{0}\right)\left(x-x_{1}\right) \ldots \ldots \ldots \ldots . .\left(x-x_{n-1}\right) f\left[x_{0}, x_{1}, \ldots \ldots \ldots \ldots x_{n-1}\right] .
\end{aligned}
$$

BMMD DATA ANALYSIS: The BMMD data were obtained with no mass (zero reading), $29.67 \mathrm{lbs}, 59.29 \mathrm{lbs}$, subject alone, subject +29.67 , and subject +59.29 . Thus, six data points are known. But, for the calibration purposes, the subject weight must be assumed to be unknown. Hence, this data point was eliminated from the data set and the interpolating polynomial of fourth order was obtained from known five data points. Ten calibration equations were obtained. Ten subjects alone also
took the readings. Their readings were substituted in the calibration curve equations to predict their masses. The following tables show the results. It is noted here that the values under column $D$, the difference, is obtained by subtracting the actual mass from the predicted mass. Whereas the percentage difference (column E) is the ratio of difference and the actual mass. The results show that the differences obtained are within the 2.25 lb limit (or within $1 \%$ ) except for the subject with 133.33 lbs when predicted from the calibration curve used from the readings taken for the subject with a 117.381 lbs . The predicted mass for this case is 134.67 lbs and the \% difference is negligibly higher than $1 \%(1.003 \%)$. Thus, it can be concluded that the interpolation technique utilizing the divided differences is a suitable tool to generate the calibration equation and subsequently, to predict the mass from the data taken for the BMMD.

## BMMD TABULATED RESULTS

|  | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |  |
| 2 | READING | ACTUAL MASS | PREDICTED MASS | DIFFERENCE | \% DIFFERENCE |
| 3 |  | (LBS) | (LBS) | (LBS) |  |
| 4 | BJECT \#1 CALIBRATION EQUATION: 157.261 LBS |  |  |  |  |
| 5 | 5.93261 | 133.33 | 134.0973028 | 0.7673028 | 0.575491487 |
| 6 | 6.76327 | 182.466 | 183.6073337 | 1.1413337 | 0.625504861 |
| 7 | 5.63984 | 117.56 | 117.9817562 | 0.4217562 | 0.358758251 |
| 8 | 6.91293 | 191.793 | 193.0788644 | 1.2858644 | 0.670443864 |
| 9 | 6.52479 | 167.926 | 168.8483943 | 0.9223943 | 0.549286174 |
| 10 | 6.59123 | 171.867 | 172.9179858 | 1.0509858 | 0.61151111 |
| 11 | 6.28027 | 153.314 | 154.1613111 | 0.8473111 | 0.552663879 |
| 12 | 6.59984 | 172.239 | 173.4477813 | 1.2087813 | 0.701804644 |
| 13 | 7.11703 | 205.025 | 206.2438464 | 1.2188464 | 0.594486721 |
| 14 | 5.58313 | 114.307 | 114.945311 | 0.638311 | 0.55841812 |
| 15 |  |  |  |  |  |
| 16 |  |  |  |  |  |
| 17 |  |  |  |  |  |
| 18 | BJECT \#2 CALIBRATION EQUATION: 159.704 LBS |  |  |  |  |
| 19 | 5.93261 | 133.33 | 134.0421966 | 0.7121966 | 0.534160804 |
| 20 | 6.76327 | 182.466 | 183.452804 | 0.986804 | 0.540815275 |
| 21 | 5.63984 | 117.56 | 117.9442557 | 0.3842557 | 0.326859221 |
| 22 | 6.91293 | 191.793 | 192.894795 | 1.101795 | 0.574470914 |
| 23 | 6.52479 | 167.926 | 168.7323413 | 0.8063413 | 0.480176566 |
| 24 | 6.59123 | 171.867 | 172.7921837 | 0.9251837 | 0.538313754 |
| 25 | 6.28027 | 153.314 | 154.0755293 | 0.7615293 | 0.496712172 |
| 26 | 6.59984 | 172.239 | 173.3206634 | 1.0816634 | 0.62800144 |
| 27 | 7.11703 | 205.025 | 206.0116712 | 0.9866712 | 0.481244336 |
| 28 | 5.58313 | 114.307 | 114.9105691 | 0.6035691 | 0.528024618 |
| 29 |  |  |  |  |  |
| 30 |  |  |  |  |  |
| 31 3JECT \#3 CALIBRATION EQUATION: 154.735 LBS | 3JECT \#3 CALIBRATION EQUATION: 154.735 LBS |  |  |  |  |
| 32 | 5.93261 | 133.33 | 133.6388295 | 0.3088295 | 0.231627916 |
| 33 | 6.76327 | 182.466 | 182.9875241 | 0.5215241 | 0.285819879 |
| 34 | 5.63984 | 117.56 | 117.5408166 | 0.0191834 | 0.016317965 |
| 35 | 6.91293 | 191.793 | 192.3844542 | 0.5914542 | 0.308381536 |
| 36 | 6.52479 | 167.926 | 168.3092092 | 0.3832092 | 0.228201231 |
| 37 | 6.59123 | 171.867 | 172.360358 | 0.493358 | 0.287058016 |
| 38 | 6.28027 | 153.314 | 153.669671 | 0.355671 | 0.231988599 |
| 39 | 6.59984 | 172.239 | 172.8875556 | 0.6485556 | 0.376543988 |
| 40 | 7.11703 | 205.025 | 205.4090667 | 0.3840667 | 0.187326765 |
| 41 | 5.58313 | 114.307 | 114.5080631 | 0.2010631 | 0.175897452 |
| 42 |  |  |  |  |  |
| 43 |  |  |  |  |  |



Page 2


|  | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 138 |  |  |  |  |  |
| 139 | READING | ACTUAL MASS | PREDICTED MASS | DIFFERENCE | \% DIFFERENCE |
| 140 |  | (LBS) | (LBS) | (LBS) |  |
| 141 JECT \#10 CALIBRATION EQUATION: 144.469 LBS | JECT \#10 CALIBRATION EQUATION: 144.469 LBS |  |  |  |  |
| 142 | 5.93261 | 133.33 | 134.449388 | 1.119388 | 0.839561989 |
| 143 | 6.76327 | 182.466 | 184.0117353 | 1.5457353 | 0.847136069 |
| 144 | 5.63984 | 117.56 | 118.2742772 | 0.7142772 | 0.607585233 |
| 145 | 6.91293 | 191.793 | 193.4573333 | 1.6643333 | 0.867775831 |
| 146 | 6.52479 | 167.926 | 169.2662149 | 1.3402149 | 0.798098508 |
| 147 | 6.59123 | 171.867 | 173.3351081 | 1.4681081 | 0.854211745 |
| 148 | 6.28027 | 153.314 | 154.5652385 | 1.2512385 | 0.816128012 |
| 149 | 6.59984 | 172.239 | 173.8646525 | 1.6256525 | 0.94383531 |
| 150 | 7.11703 | 205.025 | 206.560188 | 1.535188 | 0.74878088 |
| 151 | 5.58313 | 114.307 | 115.2255601 | 0.9185601 | 0.803590419 |
| 152 |  |  |  |  |  |
| 153 |  |  |  |  |  |
| 154 |  |  |  |  |  |
| 155 |  |  |  |  |  |
| 156 |  |  |  |  |  |
| 157 |  |  |  |  |  |
| 158 |  |  |  |  |  |
| 159 |  |  |  |  |  |
| 160 |  |  |  |  |  |
| 161 |  |  |  |  |  |
| 162 |  |  |  |  |  |
| 163 |  |  |  |  |  |
| 164 |  |  |  |  |  |
| 165 |  |  |  |  |  |
| 166 |  |  |  |  |  |
| 167 |  |  |  |  |  |
| 168 |  |  |  |  |  |
| 169 |  |  |  |  |  |
| 170 |  |  |  |  |  |
| 171 |  |  |  |  |  |
| 172 |  |  |  |  |  |
| 173 |  |  |  |  |  |
| 174 |  |  |  |  |  |
| 175 |  |  |  |  |  |
| 176 |  |  |  |  |  |
| 177 |  |  |  |  |  |
| 178 |  |  |  |  |  |
| 179 |  |  |  |  |  |
| 180 |  |  |  |  |  |
| 181 |  |  |  |  |  |

## BLIND STUDY CAL EQUATION CURVES (TEN)



## CALIBRATION EQUATIONS FROM BLIND STUDY (OCT ‘92)

## CAL EQN BASED ON SUBJECT\#1:

$$
\begin{aligned}
y= & (x-2.70593) * 30.88566+(x-2.70593)(x-3.66657) * 4.752144 \\
& +(x-2.70593)(x-3.66657)(x-4.42499) *(-0.04981263) \\
& +(x-2.70593)(x-3.66657)(x-4.42499)(x-6.83242) *(-0.024357993)
\end{aligned}
$$

CAL EQN. BASED ON SUBJECT\#2:

$$
\begin{aligned}
y= & (x-2.70602) * 30.89723831+(x-2.70602)(x-3.66630) * 4.736262145 \\
& +(x-2.70602)(x-3.66630)(x-4.42503) *(-0.051263035) \\
& +(x-2.70602)(x-3.66630)(x-4.42503)(x-6.875512) *(-0.027024517)
\end{aligned}
$$

CAL EQN. BASED ON SUBJECT \# 3:

$$
\begin{aligned}
y= & (x-2.706044)^{*} 30.89315259+(x-2.706044)(x-3.666451) * 4.56652349 \\
& +(x-2.706044)(x-3.666451)(x-4.430486) *(0.007410326) \\
& +(x-2.706044)(x-3.666451)(x-4.430486)(x-6.7989925)^{*}(-0.05134891)
\end{aligned}
$$

CAL EQN. BASED ON SUBJECT \# 4:

$$
\begin{aligned}
y= & (x-2.70577) * 30.91107986+(x-2.70577)(x-3.66562) * 4.759190578 \\
& +(x-2.70577)(x-3.66562)(x-4.42344) *(-0.045210185) \\
& +(x-2.70577)(x-3.66562)(x-4.42344)(x-5.99615) *(-0.018736277)
\end{aligned}
$$

CAL EQN. BASED ON SUBJECT \# 5:

$$
\begin{aligned}
y= & (x-2.70629) * 30.89916894+(x-2.70629)(x-3.66651) * 4.741844172 \\
& +(x-2.70629)(x-3.66651)(x-4.42504)((-0.0417715281) \\
& +(x-2.70629)(x-3.66651)(x-4.42504)(x-6.5493) *(-0.026655905)
\end{aligned}
$$

CAL EQN. BASED ON SUBJECT \# 6:

$$
\begin{aligned}
y= & (x-2.70628) * 30.90431848+(x-2.70628)(x-3.66634) * 4.740597853 \\
& +(x-2.70628)(x-3.66634)(x-4.42483)(-0.046780969) \\
& +(x-2.70628)(x-3.66634)(x-4.42483)(x-6.37053) *(-0.019335521)
\end{aligned}
$$

CAL EQN. BASED ON SUBJECT \# 7:

$$
\begin{aligned}
y= & (x-2.70581) * 30.91558908+(x-2.70581)(x-3.66552) * 4.710170103 \\
& +(x-2.70581)(x-3.66552)(x-4.42477) *(-0.033932645) \\
& +(x-2.70581)(x-3.66552)(x-4.42477)(x-6.58906) *(-0.030577471)
\end{aligned}
$$

CAL EQN. BASED ON SUBJECT \# 8:

$$
\begin{aligned}
y= & (x-2.70576) * 30.90689389+(x-2.70576)(x-3.66574) * 4.763891327 \\
& +(x-2.70576)(x-3.66574)(x-4.42348) *(-0.027091221) \\
& +(x-2.70576)(x-3.66574)(x-4.42348)(x-6.02807) *(-0.054254941)
\end{aligned}
$$

CAL EQN. BASED ON SUBJECT \# 9:

$$
\begin{aligned}
y=(x & -2.70590) * 30.91204601+(x-2.70590)(x-3.66572) * 4.754776284 \\
& +(x-2.70590)(x-3.66572)(x-4.42366) *(0.006299436) \\
& +(x-2.70590)(x-3.66572)(x-4.42366)(x-6.17017) *(-0.081598626)
\end{aligned}
$$

CAL EQN. BASED SUBJECT \# 10:

$$
\begin{aligned}
& y=(x-270587) * 30.91494483+(x-2.70587)(x-3.66560) * 4.762522147 \\
&+(x-2.70587)(x-3.66560)(x-4.42326) *(-0.038329713) \\
&+(x-2.70587)(x-3.66560)(x-4.42326)(x-6.621904) *(-0.036791118)
\end{aligned}
$$

NOTE: $y=$ predicted mass and $x=B M M D$ reading
This blind study data were collected during the month of October (1992) Ten subjects worked with the cal weights and obtained readings to help generate ten cal equations. Additional ten subjects obtained readings by themselves only. Their readings were substituted in each of the ten cal equations to predict their masses. The comparison was made between the predicted masses and their scale weights (See report on BLIND STUDY DATA analysis)

