Methods of Mass Meas -

1. Theoretical
2. Exp - Limits
   Metrology
   Imposed by environment
3. Possible application -

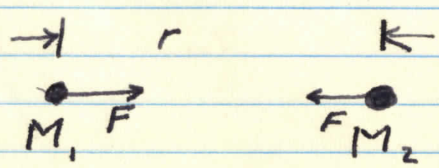
1.1 $F = MA$ $\qquad M = \frac{F}{A}$ $\qquad$ - ratios

.2 $T = \sqrt{M/K}$

.3 $F = \frac{M_1 M_2}{r^2}$

Although mass $\bar{c}$ length and time form the triad of fundamental physical quantities by which all other physical quantities are derived, it does not lend itself to direct determination as for example length- the simplest and almost universal method of mass determination is by weighing which is ~~a compar~~ basically a ratio measurement under a ubiquitous gravitational field. Under so weightless conditions ~~this~~ mass must be measured in terms of other quantities - Some of the possible quantities + arrangements for this measurement will be described.

One of the properties of all mass is that it develops a field of mutual attraction for all other mass as follows -



M_1 = Mass of 1st Obj.
M_2 = " " 2° Obj.
r = distance between objs.

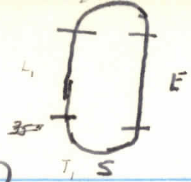
$$F = k\frac{M_1 M_2}{r^2}$$

This force of mutual attraction or gravity only becomes appreciable when one of the masses is large or when the distance r is very small. The ~~great~~ mass of a planet such as earth remains relatively constant and develops an appreciable accelerative force which provides the convenient means of "weighing" a mass, usually through comparison of the forces produced by known & unknown masses by means of ~~the balance or~~ lever ~~scales~~ arms (balance scales) or more commonly by measurement of the ~~calibrated~~ deflection produced by the gravitational force unknown mass on a calibrated compliance - (spring scales) -

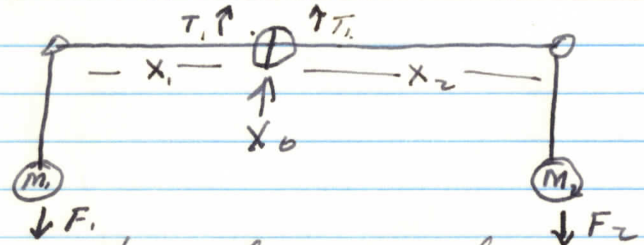
It may be worth the effort to examine these current universal methods of mass measurement in some detail. The earth's mass of some tons produces an attractive or accelerative force everywhere at its surface. The value of this ~~form~~ acceleration is the same for all ~~equal~~ masses as shown by Galileo + numerous others and nominally is some 32 feet per sec per sec. i.e. ~~an object~~ a frictionless object dropped will be travelling at a velocity of 32 feet per second at the end of the first sec. and ~~so~~ will add 32 feet per sec. to this velocity for every additional ~~fo~~ second which it ~~accelerates~~ falls. -

This acceleration, A, then produces a force, F, on every mass, M, ~~under subje~~ which is directly proportional to this mass. $F = MA$. This force F is the weight which ~~we ar~~ we ~~are accustomed~~ experience. For a given location on earth this force varies c̄ the mass of the objects. Some stable object may be designated a "standard" and ~~other~~ the mass of other objects determined in terms of this standard. The international standard is a platinum kilogram kept

T_1 - 1'35"
L_1 - 1'40"
T_2 - 1'35
L_2 -



in Paris. Other masses may be compared to this by means of lever arms of known length.



In order for a beam, free to rotate about some point X_0 on its axis, to remain stable the torque T_1 must exactly equal T_2. Torque T is equal to the product of distance of the applied force X. multiplied by the value of the force F. The products of the ~~$M_1 F_1$~~ force-distances must then be equal.

$\cancel{M_1} X_1 F_1 = \cancel{M_2} F_2 X_2$ — Since the Forces are directly proportional to mass we may then replace as follows:

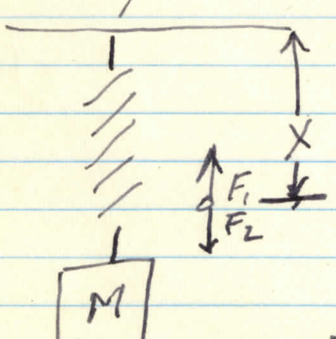
$M_1 X_1 = M_2 X_2$

If one mass and the distances at balance are known then ~~the~~ an unknown mass ~~M~~ may be determined from

$$M_x = \frac{M_1 X_2}{X_2}$$

In a common ~~an~~ alternative arrangement the known mass is moved along the arm i.e. X_1 is varied until a condition of balance is obtained. Under these conditions X_1 may be calibrated directly in terms of the unknown mass. There are many variations of this basic balance scheme.

A second and widely used scale depends upon deflection of a calibrated compliance, usually a spring or torsion bar by the unknown force. ~~The~~ In a position of equilibrium the restoring force of the spring F_1 must equal the force produced by the wt. F_2 the spring



force is given by

$F_1 = KX$ ~~while and~~

~~the~~ thus $MA = KX$

or mass may be expressed in terms of deflection of the spring $M = X\left(\frac{K}{A}\right)$

The majority of the commercial scales in this country are variations of this basic spring scale. The heart of this scale is the spring for its stability & linearity will determine the performance of the scale so that development of such springs have reached a rather advanced state.

For convenience some of the quantities involved in gravimetric determination of mass will be given. The first such quantity is the gravitational constant of earth. It is in fact constant only at a given location and for restricted periods of time. The variation c̄ location is significant, for example from

distance of to mi. there is a difference of The balance scale will not be affected by this but the uncorrected spring scale would suffer ~~an error~~ a difference of between the two locations.

In the English system typical values for G are —

In the metric cm gm sec system this becomes ______ -

In the English system the unit of mass is the slug which under a G of ——— produces a force of ——— #. Conversely pounds may be converted to mass if the ~~In the metric system~~ acceleration constant is known -

$$M = \frac{F}{A} \quad \text{Mass (slugs)} = \text{Force} \left(\frac{\text{pounds "weight"}}{\text{acceleration}} \right)$$

It is obvious that the attractive forces produced by the small masses involved in current satellites will be to small & irregular to be useful in determination of mass under weightlessness.

A 2° possibility for determination of mass by means of the Newtonian equation.

Force = Mass x Acceleration