Mass

Exp

#1

7530-286-6951

FEDERAL SUPPLY SERVICE

16—74548-1 GPO

Design

Spring Force = KX

$F = K X$ $\quad K = \frac{F\,(\#)}{X\,(in)}$

$T = 2\pi\sqrt{\frac{m}{k}}$ & $T^2 = 4\pi^2 \frac{m}{k}$

$1 = 2\pi$

$K = \frac{4\pi^2 m}{T^2}$ $\bar{c}$ $T = 1$

$F = ma$

$K\left(\frac{F}{X}\right) = \frac{4\pi^2 m}{sec^2} = \frac{F}{\cancel{X\,sec^2 \cdot \frac{dx}{sec^2}}}$ $\quad m = \frac{F}{a} = \frac{F}{\frac{dx}{dt^2}\,sec}$

$\bar{c}$ m = [illegible] $\frac{170^{\#}}{32}$ 39.

644
644
708^4

$Acc = 32.2\ ft/sec^2$

$= 32.2 \times 12''/\cancel{ft} \times ft/sec^2$

$= 708,4\ ''/sec^2$

From H.B. Vib. P-1-13
Harris

Calculate spring constants:

$$\text{Weight} = \text{Force}_{(\#)} = \text{Mass} \times \text{Acc.}$$

$$\times \text{ in/sec}^2$$

$$\& \text{ Mass} = \frac{\text{Force}}{\text{Acc}} = \frac{\#}{\text{in/sec}^2}$$

$$= \# \times \frac{\text{sec}^2}{\text{in}}$$

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$2\pi f = \sqrt{\frac{K}{m}}$$

$$\{4\pi\}^2 f^2 = \frac{K}{M}$$

$$K = 4\pi^2 f^2 \cdot m \qquad 4\pi^2 = 4 \cdot 9.86$$

$$= 39.4$$

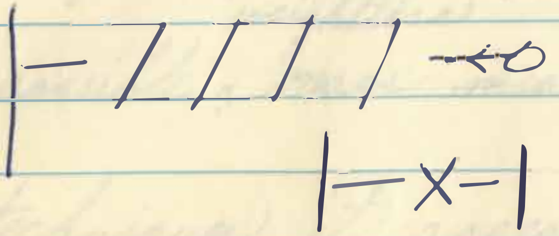
f=1

$$= 39.4 \cdot 1 \cdot 170 / 108.4$$

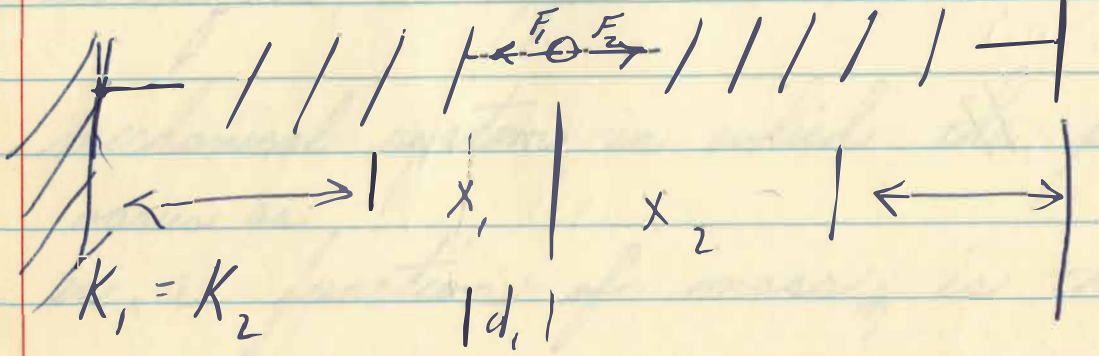
$$= 39.4 \quad .226$$

$$= 8.9.3 \ \#/\text{in}$$

Springs in series -



$F = -KX$



$$-K_1X_1 + K_2X_2$$

$$F = F_1 - F_2 = -K_1 X_1 - (-K_2 X_2)$$

$$= -K(X_1 - X_2)$$

At rest $X_1 = X_2$

$$F = -K\left[(X_1 - d) - (X_2 + d)\right]$$

Example - $K = 1$

$X_1 = 2 \quad X_2 = 4$

$$F = -1(2-4) = (-2)$$

This is equiv to doubling K — 2K

Mass determination by oscillating theoretical spring system

After study of the proposed and possible "weightless" mass ~~measuring~~ measurement techniques, it appears that an oscillating mechanical system in which ~~the~~ period ~~is~~ varies as a function of mass, is the most promising. ~~This takes into account~~ the practicality & accuracy of various measuring techniques, ~~constrain~~ accuracy desired, limits imposed by the characteristics of the human body to be measured and the constraints imposed

have all been considered in this conclusion
by the conditions of measurement.
which ~~unanimously~~ shared
~~This conclusion~~ was ~~shared~~ by
contacted
a number of people who have
theoretical and/or practical experience
in the field including ~~the Chief~~
Mr. Mc Nish Chief Metrology Div. N.B.S.,
Dr. Everett Palmatier Chairman Dept. Physics
Frank Riley
U.N.C., Mr. ~~Byron~~ Proj. Eng. of
Lockheed A/C who ~~he~~ did preliminary
work in the area, and Dr. Ritter,
Physicist AMD. A second

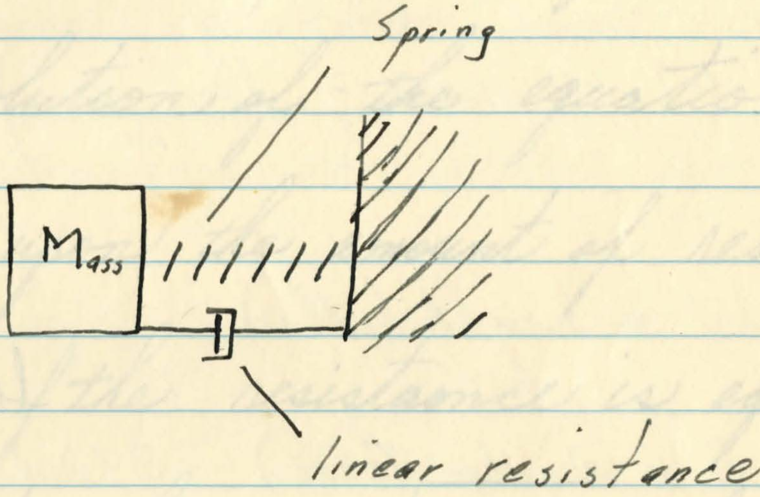
* It should be pointed out that the Lockheed demonstration of this method, was ~~a very crude first attempt~~ of necessity extremely crude, ~~in the absence of~~ apparently for lack of material support. Unfortunately the idea is extant that ~~this represented~~ ~~a reas~~ the results represented are what ~~what~~ might be reasonably expected from such a system. Mr. ~~Albright~~ Riley, the project engineer is well aware of this and feels that the required accuracies may be obtained in flight &/or demonstrated on earth c̄ adequate instrumentation

unanimous opinion is that little further is to be gained from theoretical studies c̄ an accompanying practical experiment(s) * that the described experiment will be a realistic demonstration of what may be expected. The problems in ~~such a system~~ ~~may be as~~ ~~proving~~ demonstrating the accuracy of such a system may be arbitrarily divided into three areas:

1) theoretical
2) constraints of experimental conditions
3) instrumental details —

The ~~first order and to some extent 2° order~~ solutions of the general differential equations of an ~~motion of a particle an~~ oscillating system ~~a~~ are ~~extremely~~ solved in ~~great~~ detail ~~well worked out~~, since the same equations describes a number of ~~py~~ physical systems including mechanical, electrical + acoustic.

The idealized system consists of a mass ~~m~~ M, ~~allow~~ attached to a restoring force F = K X, provided by a massless spring attached to a ridgid support. Motion of the particle ~~will~~ is assumed to create a linear resisting force of ~~resistance~~ $m R' \frac{dx}{dt}$.



The general equation of motion of such a system undergoing "natural" oscillation (i.e. displaced from its position of rest & allowed to return s̄ outside influence) is:

1.1 $$M \frac{d^2X}{dt^2} + R\frac{dX}{dt} + KX = 0$$

There are two possible forms to the solution of the equation depending upon the amount of resistance present. If the resistaance is equal or greater than $2\sqrt{KM}$ then the mass will return to the position of equilibrium in an exponentional fashion. If however, as the case will be here, $\frac{R}{2\sqrt{KM}} < 1$

1.1 $F_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{K\cancel{g}G}{W}}$

Where

F_n = undamped natural frequency (~~cycles~~ C.P.S.)

K = spring constant (lbs./in)

M = mass $\left(\frac{\text{lbs. sec}^2}{\text{in}}\right) = \frac{W\ (\text{lbs.})}{G\ (\text{ins./sec}^2)}$

W = weight (lbs.)

G (Austin Texas) 979.283 cm/sec²

= 385.54372 in/sec²

$1''_{(U.S.)}$ = 2.540005 cm.

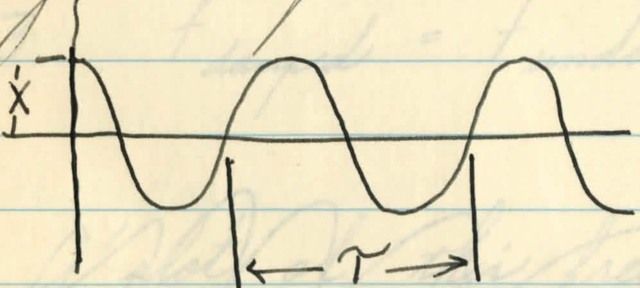
And - $F_n = \frac{1}{T_n}$

1.2 $T_n = \frac{1}{2\pi}\sqrt{\frac{W}{K\cancel{g}G}}$

3 2.2
1
644
512
386.4

~~then~~ an oscillation about the equilibrium point will result. If the resistance is zero, an undamped or continuous oscillation will occur whose frequency is given by:

1.1 $f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}$ and whose amplitude will be equal to the amplitude of the original displacement.



L.v. Space -

Undamped natural oscillation

~~This frequency is called the undamped natural frequency.~~

If ~~on as ano~~ the resistance is not zero, i.e. energy is dissapated as in any ~~real~~ practical system, both the frequency and amplitude will be modified from the undamped case.

The frequency will be shifted by:

$W = 2\pi F$ ~~ω_{damped}~~

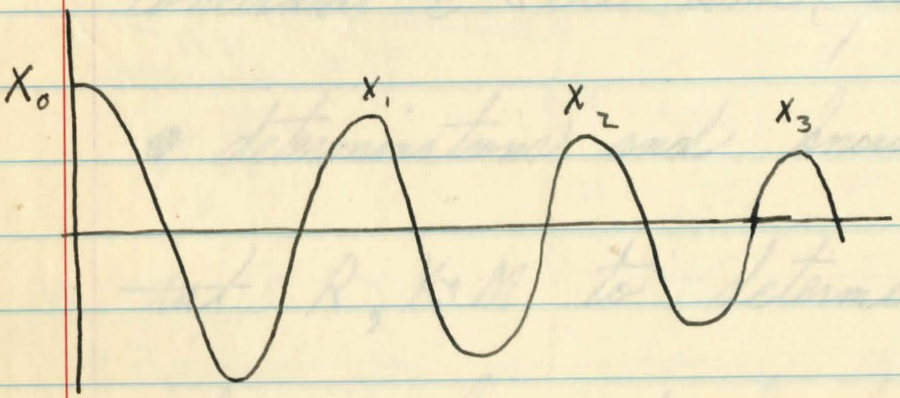
$$1.3 \quad F_{\text{damped}} = F_{\text{undamped}} \left(1 - \left[\frac{R}{2\sqrt{KM}} \right]^2 \right)$$

A plot of this frequency shift is shown in (— 2v. 3" x 5" blank

(——) ~~Pg. 2-4 Shock & Vib. HB.~~

In addition the peak amplitude will decrease by

$$\frac{X_n}{X_0} = e^{-2\pi n \frac{R}{2\sqrt{KM}} / \sqrt{1-\left(\frac{R}{2\sqrt{KM}}\right)^2}}$$



~~A p~~ The relationship of three successive peak amplitudes to the "damping ratio" is shown in .

L V 4"X4" space (pg 2-5 5+V HB)

Reasonable
*/; Variations in amplitude will be of no concern if the time t_0 is determined from zero crossings.

The two fundamental questions that arise from ~~the~~ purely theoretical considerations then are concerned c̄ the time (frequency) determinations and knowledge of ~~that~~ R, K+M to determine deviations from the natural frequency - insert ① here

~~Considering time resolution required:~~

$$\cancel{f = \frac{1}{2\pi}\sqrt{\frac{K}{M}}} \qquad \cancel{f = \frac{1}{T}}$$

$$F \propto \sqrt{\frac{K}{M}} \qquad T \propto \frac{1}{\sqrt{\frac{K}{M}}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{Kg}{W}}$$

$$f_n^2 = \frac{1}{4\pi^2}\,\frac{Kg}{W}$$

$$\left(\frac{1}{T}\right)^2$$

$f = \frac{1}{\;}$

$$\frac{x}{\cancel{T}} = \frac{1}{2\pi}\sqrt{*}$$

$$\frac{T}{2\pi} = \frac{1}{\sqrt{*}}$$

$$\tau = \frac{2\pi}{\sqrt{*}}$$

$$\tau^2 = \frac{4\pi^2}{\frac{Kg}{W}}$$

$$\tau^2 = \frac{4\pi^2 W}{Kg}$$

$$\tau = 2\pi\sqrt{\frac{W}{Kg}}$$

32.2
12
644
322
3864

3.864×10^2

Example:

$$\tau_1 = 2\pi\sqrt{\frac{200}{10\cdot 386.4}}$$

$$= 2\cdot 3.1416\sqrt{\frac{20}{386.4}}$$

$$= 6.2832\sqrt{\frac{20\times10^{-2}}{3.864}}$$

$5.1\cancel{8}759834$

3.864) 20.000,00
19320
6800
3864
29360

$\frac{2000}{3864}$

5.17857

$\sqrt{5.17857} \times 10^{-1}$

~~71483/2~~

$\frac{71399}{2} = 35699$

35708

$\frac{71416}{2}$

The minimum time resolution ΔT required to measure a mass difference of ΔM ~~is~~ of ~~.001~~ (10^{-3}) is: $\Delta T = \frac{1}{\sqrt{\frac{1}{10^{-3}}}} = \sqrt{10^3}$

$$\approx \frac{1}{31.7} \approx .031$$

~~Thus for a change in mass of .1%~~ ~~at 150#~~ ~~a time resolution of~~ ~~0~~ [illegible] ½ millisec. ~~is~~ ~~required~~. If the period is ~~0~~ say one sec. $\bar{c}$ a time resolution of ~~30~~ ½ msec is required. This is easily achieved by a counter of ~~100~~ 10^4 c.p.s.

* It will be shown later that this time measurement will in reality be dependent upon a distance measurement.

Microsecond counters are standard today and fractional millisec counters which will completely eliminate practical errors in timing here are relatively simple & ~~crude~~ devices -

that could not be achieved by a stop watch or other crude means. *

¶ The next consideration, effects of resistance, cannot be determined ē such accuracy - The resistance in the ~~model case~~ real ~~[illegible]~~ case will not be lumped but will consist of several components most of which do not lend themselves to calculation. The two chief sources of resistance arise from: 1) the motion of the

* While the spring resistance will remain a constant, the viscous resistance (air drag) will vary c̄ subj. configuration and surrounding atmosphere + cause variable deviations from the undamped natural frequency.

(man)
mass through a viscous medium (air).
& 2) the dissipation of the spring. – *
Since the body will be irregular
& variable and the ~~Reynolds #~~ ~~is~~
~~unknown~~ atmosphere is presently
unknown only ~~an~~ ~~a most crude~~ ~~estimation~~ could ~~may be~~
~~made~~ flat plate ~~Area of seated figure mobile~~
~~ant. & post. assumed to be~~
~~[illegible]~~
an estimation may be made of
this effect.

Air "drag" at ~~more~~ ~~usual~~
A/C velocities may be calculated

Omit

Hysteresis : $F \; Q = R \frac{dx}{dy}$

Work $= F \times D = R \frac{dx}{dy} \cdot D$

Hysteresis $= .02 \% = 2 \times 10^{-4} =$

$\frac{\Delta X}{X}$ or $\frac{\Delta D}{D}$

At ~~a~~ "rate" of 5#/" & 1" displacement this is

~~equivalent to~~ will require $5^{\#} \times 2 \times 10^{-4}$

$\times 1 = 10^{-3}$ #. For practical purposes

the work done will then be equivalent

!! ~~5#~~ 10^{-3} # $\times 2 \times 10^{-4}$ " $= 2 \times 10^{-7}$ ___

~~Aerodynamic Drag~~

~~The second factor is energy dissipated in the spring which is expressed in terms of hysteresis~~

from: $D = C_d \; A$ [illegible] P

P = dynamic press. = $\frac{\rho V^2}{2}$

D = drag

C_d = Coeffecient of drag

= 1.28 for flat plate

ρ = density = 2.38×10^{-3}

for normal atmosphere

V = Velocity (ft/sec)

(over on back)

Resistance $R = \frac{Force}{Velocity}$ -

and combining this c̄ the drag force -

$$R = \frac{C_d A \rho V}{2}$$

Assuming a flat plate area of 3 ft.^{2} for a seated man in a normal atmosphere c̄ a velocity of 1 ft./sec. (this is greater than our maximum), the drag is calculated to be

$$R = \frac{1.28 \times 3 \times 2.38 \times 10^{-3} \times 1}{2} = 4.57 \times 10^{-3}$$

#/ft/sec -

~~Rather than attempting guessing at all Resistance quantities~~ ~~An alternative~~

We may ~~is to~~ note the maxiamum allowable effects of resistance for the accuracies desired (auxillary calibration is possible but undesiable).

~~For small values of $R/2\sqrt{KM}$ (< .1)~~

~~O~~ We have seen that a tolerance of ~~±1.5%~~ ~.5 msec. is allowable ~~for~~ in time for an accuracy of ±.1% in mass determination at 150#.

insert ②

$\omega = \cancel{2\pi f}$

Omit

$$F_d = F_{ud}\left(1 - \overset{k}{\left(\frac{R}{2\sqrt{KM}}\right)^2}\right)^{1/2} \qquad F = \frac{1}{T}$$

$$\frac{1}{T_d} = \frac{1}{T_{ud}}\left[1 - \overset{f}{(K)^2}\right]^{1/2}$$

$$\frac{T_{ud}}{T_d} = \left[1 - (K^2)\right]^{1/2}$$

$$\frac{T_{ud}}{T_d(c)} = Td(c)$$

$$\frac{1}{c} = \frac{Td}{Tud}$$

$$\frac{1}{[1-K^2]^{1/2}} = \frac{Td}{Tud}$$

$$[1-K^2]^{-1/2} =$$

$$+K^2 \left(\frac{T_{ud}}{T_d}\right)^2 = 1 - (K)^2 + K^2$$

$$K^2 = 1 - \left(\frac{T_{ud}}{T_d}\right)^2$$

$$K = \sqrt{1 - \left(\frac{T_{ud}}{T_d}\right)^2}$$

$$= \cancel{\sqrt{1 - (01.5 \times 10^{-2})^2}}$$

$$= \cancel{\sqrt{1 - 2.25 \times 10^{-4}}}$$

log 984

.99300

1986^{2}

$\frac{1}{1.015} \approx .984$

$$\begin{array}{r} 1.000 \\ .967 \\ \hline .033 \end{array}$$

$(.984)^2 \approx .967$

$\sqrt{.033} = \sqrt{3.3} \times \sqrt{10^{-2}}$

$f = \frac{1}{T} \approx 1.7 \times 10^{-1}$

$\approx .17$

~~ω~~ $\frac{F_d}{f_{ud}} = \frac{T_{ud}}{T_d} = \frac{9}{.984}$

16

T

.984

Omit

$$K = \sqrt{1 - \left(\frac{T_{od}}{T_d}\right)^2}$$

$$= \sqrt{1 - \left(\frac{1.000}{1.015}\right)^2}$$

From the chart ~~on~~ values of $\frac{R}{2\sqrt{KM}}$ of $< .1$

Values of F_o/F_{od} have been plotted for small values $\frac{R}{2\sqrt{KM}}$

f	f^2	$1-f^2$	$(1-f^2)^{1/2}$
.001 (10^{-3})	10^{-6}	.999999	
.002	4×10^{-6}	.999996	
.004	16×10^{-6}		
.008	64×10^{-6}		
.01	10^{-4}		

By examination of the chart

it is evident that large changes in

~~freq~~ the damping ratio $\left(\frac{R}{2\sqrt{K M}}\right)$ may occur

c̄ relatively little shifts in frequency

for damping ratios of say ~~0~~ .1-

The calculated damping ratio

for a period change of ~~±.5%~~

is ≈ .17 - In ~~terms of~~

units ~~this is~~ c̄ typical

conditions of:

32.2
12
644
22
864

$M = \cancel{170}\ 200\ \# / \overset{386.4}{\cancel{32.2}}\ in/sec^2 \simeq 7728. \frac{\# sec^2}{in}$

$K = 20\ \#/in$

~~Mass~~ in Error! Shouldbe

~~200~~ $\frac{2.\times10^2}{.386\times10^3} = \frac{.518\times10^{-1}}{R} = .518$

~~#6~~ $.17 = \frac{K}{2\sqrt{7.73\times10^3 \frac{\#\ \cancel{in}\,sec^2}{\cancel{sec^2}\ in} \cdot 2.0\times10\ \#/in}}$

$.17\text{-}R = \frac{1}{2\sqrt{7.73\times2}\ \times10^4\ \frac{sec^2\ \#^2}{in^2}}$

R=.17K

$.17 = \frac{R}{K}$

$R = .17\times\frac{~~[illegible]~~}{.34\sqrt{15.46}\ \times10^2\ \frac{sec\cdot\#}{in}}$

$= \frac{~~[illegible]~~}{.34\cdot3.94\ \times10^2}$

$= \frac{}{1.34\ \times10^2}$

$R = .17\times.134 = 23.8\ \#/in/sec$

~~This is~~

$R = .17\sqrt{.518\ \frac{\#\ sec^2}{in}\times20\ \#/in} = .34\sqrt{10.36\ \frac{\#^2 sec^2}{in^2}}$

$= .34\times3.22 = \quad \#/in/sec$

$= 1.06\ \#/in/sec.$

(follows ②)

In more ordinary terms this would be equal to ≈ 12 # wind resistance at 1 ft./sec. ~~or~~ to compare it to a realistic situation Such a value ~~is~~ obviously will not be encountered. ~~but~~ the actual value may be determined by experiment but in any event will be much less than the max. allowable amount of damping -

→ One is now faced c̄ the

Start after ②

design of an experiment under the constraints ~~impose~~ imposed by the conditions but which will also provide realistic simulation of weightlessness under gravity.

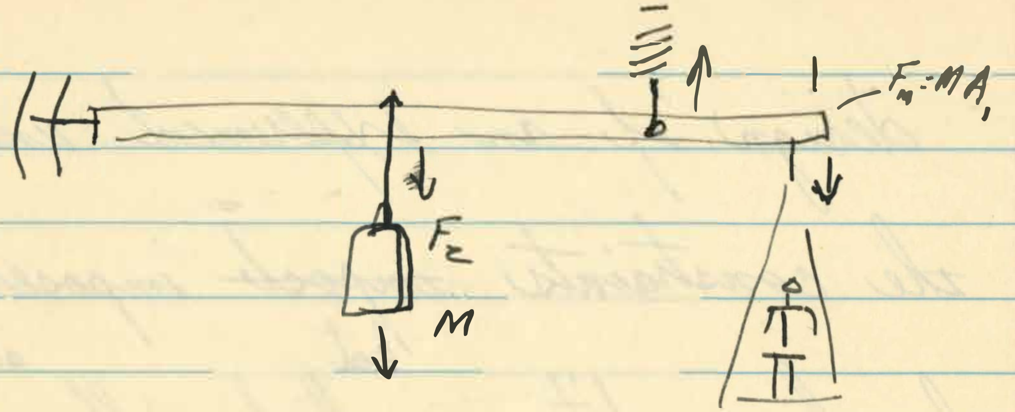
The problem may again be considered in three main areas -

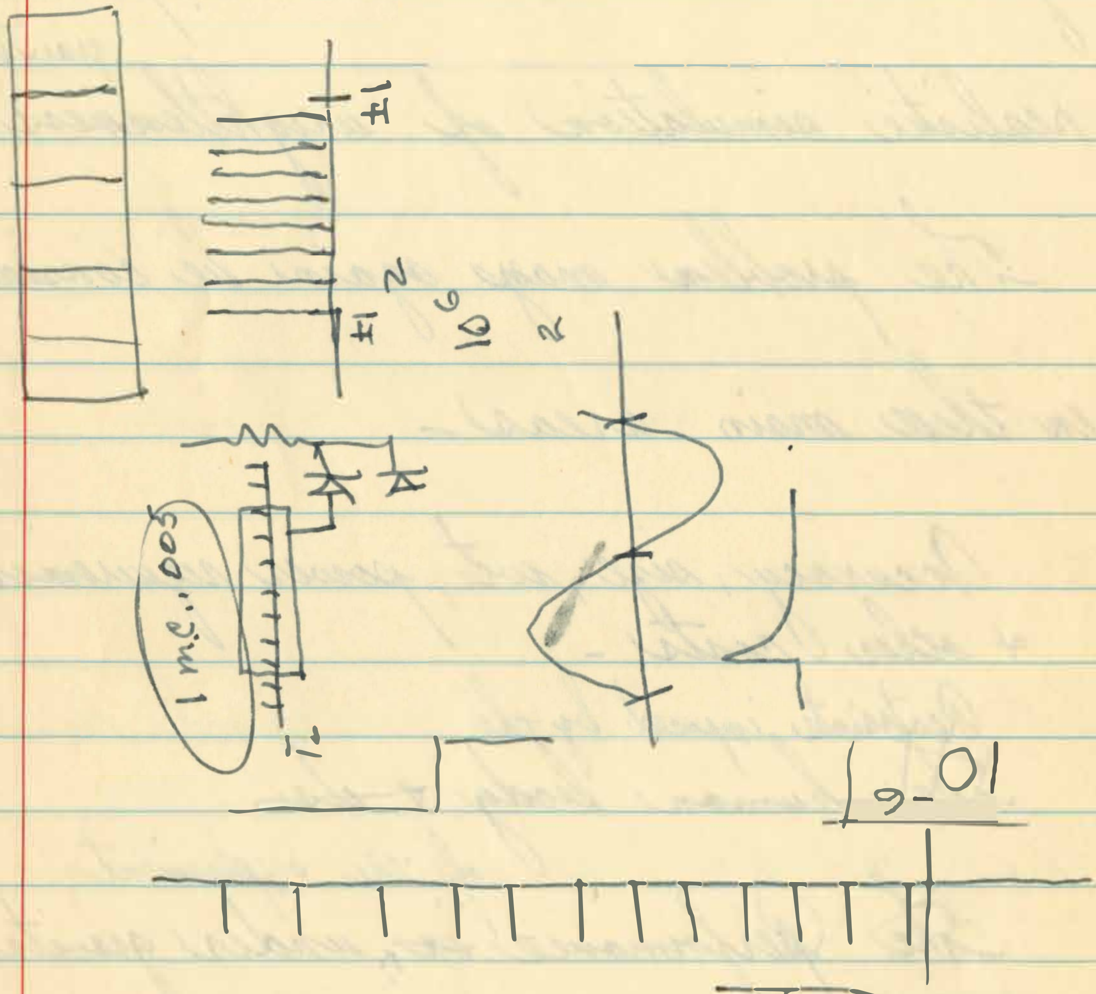
Accuracy, size, wt., power requirements & other costs -

~~The~~ Restraints imposed by the human body ~~& its~~

The performance ~~in~~ of the experiment under gravitational conditions

10^{-5}





$$M \propto \frac{T_2 - T_1}{(\Delta L)^2\, T_2 \cdot T_1}$$

ΔT