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METHODS OF DETERMINING MASS OR WEIGHT IN A ZERO-G ENVIRONMENT

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ABSTRACT

Four methods of determining the mass or earth weight in zero-g, non-rotating space laboratories are investigated in this paper. For the want of more descriptive names these four methods may be classified as:

1. Energy-Velocity
2. Vibrating Spring and Masses
3. Centrifuge
4. Momentum

Of the four, only one, the momentum method is recommended for weighting non-rigid masses, such as man, in a zero-g environment. This is because of the high accuracy required in laboratory work. All of the first three methods possess similar characteristics, which lead to inaccuracies, namely re-arrangement of the soft mass under accelerating forces. In the first method this results in indeterminant energy losses; in the second method, in unknown damping effects; and in the centrifuge method, in an indeterminant center of mass shift from a doubtful original location. Tests may prove these inaccuracies to be exaggerated; unfortunately only their existence and trend may be shown analytically. The fourth method, however, provides a way around these inaccuracies. It utilizes the principle of momentum conservation, which by-passes energy losses.

Realistic numbers have been assigned to the centrifuge and momentum methods for the purpose of determining accuracies. These results are shown in Figures 4 and 6.

INTRODUCTION

The next step in the exploration of space will include programs which provide manned orbital laboratories. One of the prime functions of these laboratories will be to determine man's physiological reactions to this weightless environment. To accomplish this will require measuring devices of many different types. It is difficult to imagine such a laboratory without an instrument for measuring the mass and mass changes of man. After all, the simplest physical examination in the doctor's office requires a weight determination. In the author's opinion, the need for scales in orbiting biological laboratories is definitely established; but these scales will not be as simple as the popular bathroom variety which operate under the constant accelerating force of gravity. Since in orbit centrifugal force completely balances the gravitational force, the net result is the well known weightless condition in which no forces appear to be acting. Therefore, to sense the magnitude of a mass, artificial forces must be provided. Rotating space stations inherently provide artificial gravitational forces which can be utilized for mass determination. The problems would be similar to the centrifuge problems covered here. However, it is assumed that the first space laboratories will be of the non-rotating type, to which this discussion is restricted.

It is felt that determining man's mass under zero-g, either because of its apparent simplicity or because of concentration on more sophisticated physiological measuring devices, has not received sufficient attention.

This paper explores four methods of utilizing accelerating forces and resulting velocities to determine the magnitude of masses, both rigid and non-rigid, the latter being representative of the human body. It is not the intention here to go into actual designs representing these methods, but rather to cover the principles and criteria which would govern possible designs.

ENERGY - VELOCITY

Determining the mass of a solid or rigid body in a zero-g field may be accomplished by converting a known amount of work, such as that stored in a cocked spring, into kinetic energies. These energies exist only in translational velocities imparted to both the specimen and the reacting space station since rotational velocity of the station is eliminated or kept to a negligible minimum by allowing the line of action to pass through or close to the c.g. of the station, a proper design feature. See Figure 1. for schematic. A simultaneous solution of the following three equations:

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ENERGY - VELOCITY SCHEMATIC

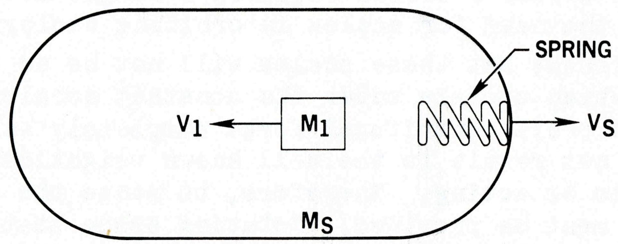

FIGURE 1

$$M_S V_S = M_1 V_1 \qquad \text{(conservation of momentum)} \qquad (1)$$

$$2E = M_S V_S^{\,2} + M_1 V_1^{\,2} \qquad \text{(conservation of energy)} \qquad (2)$$

$$V_M = V_S + V_1 \qquad \text{(relative measured velocity)} \qquad (3)$$

renders:

$$M_1 = \frac{M_S\, 2E}{V_M^{\,2} M_S - 2E} \qquad (4)$$

where:

E = stored energy in spring

M_1 = mass of the specimen to be determined

M_S = mass of the station

V_1 = velocity of M_1 after spring release

V_S = velocity of station after spring release

V_M = measured relative velocity between V_1 and V_S .

To determine the sensitivity of the M_1 error to errors in inputs, an analysis was performed on equation (4) using the appropriate substitutions of $V_m = s/t$ where s and t are a measured time and a measured distance, respectively. The following sensitivity equations resulted:

$$\Delta M_1 = - \left[\frac{2E}{V_M^2 M_S - 2E} \right]^2 \Delta M_S \qquad (5)$$

$$\Delta M_1 = 2 \left[\frac{V_M M_S}{V_M^2 M_S - 2E} \right]^2 \Delta E \qquad (6)$$

$$\Delta M_1 = \frac{4 M_S E s}{\left[V_M^2 M_S - 2E \right]^2} \Delta s \qquad (7)$$

$$\Delta M_1 = \frac{4 M_s E t}{\left[V_M^2 M_S - 2E \right]^2} \Delta t \qquad (8)$$

If this same principle of converting energy to velocity is applied to determining the mass or weight of man, an error is introduced because of man's non-rigidness. The blood, body fluids, organs, etc., tend to move opposite to the accelerating force and thus absorb part of the work. All have, no doubt, experienced the blood leaving the head with a sudden rise from the floor. This energy absorption would appear as an unknown term in the right hand side of equation (2); thus making simultaneous solution of the three equations impossible. Some have suggested that it might be impossible to compensate for this loss of energy by subtracting a predetermined amount of energy from the original energy. The uncertainties involved make this suggestion too risky. The absorbed energies would no doubt vary from individual to individual and could possibly vary within one individual over a period of time in either a normal or weightless condition. Figure 2 is a simple demonstration, which shows how the resulting final velocities of equal masses accelerated by the same spring can differ, thus reflecting the internal energy losses, when all portions of the mass do not act as a unit. Figure 2(b) and (c) are somewhat analogous to the human body. Clay with a coefficient of restitution of one was used to absorb the internal energy swiftly. With a spring (which represents the other extreme in restitution) between the two masses M_1 and M_2, the results would have been the same; it would have just taken longer for the oscillations to die out. It should also be noted that relative positions of M_1 and M_2 do not influence the final resulting velocity as long as there is no relative velocity between them. Granted this analogy is a poor representation of the human body; it does show that there

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ENERGY LOSS DEMONSTRATION

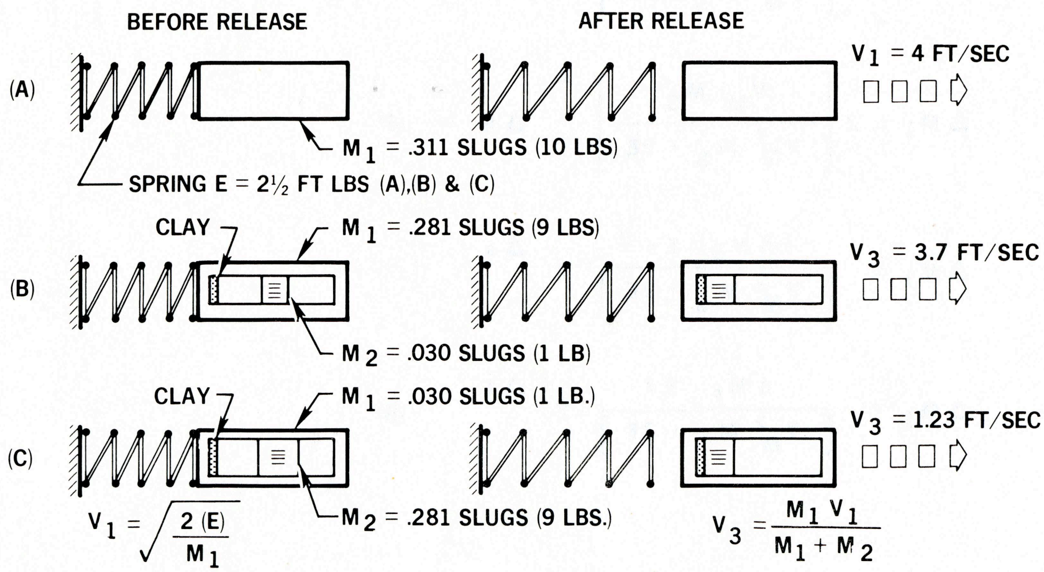

FIGURE 2

is an indeterminant energy loss when a non rigid body is accelerated. This energy degradation cast doubt on the accuracy of any system using this principle for weighing non rigid masses.

VIBRATING SPRING AND MASSES

Another type of mass or weight measuring instrument might follow some form of the vibrating spring and masses arrangement as shown in Figure 3. By simply determining the period of oscillation, T, and by knowing the spring rate, K, and the station mass, M_S; the mass, M_1, of a rigid body may be calculated from equation (11) which is a rearrangement of the period equation of 2 masses connected by a spring. A device based on this principle might provide the simplest and most accurate space scales for small rigid masses because of two features. One, the period is independent of the amplitude; and two, the accuracy of the period may be increased by determining the frequency over a longer increment of time. The main degrading factors are the energy loss to internal friction in the spring and aerodynamic damping, both of which may be kept small. Depending on the relative masses of the specimen and the spring, it may be necessary to deduct the effective mass of the spring to obtain the desired accuracy for small masses. An error analysis of the basic equation gives relationships (12), (13) and (14). From (12) it may be seen that the percentage of error in the spring constant is reflected almost directly in

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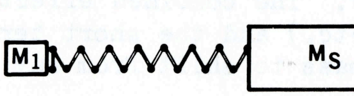
VIBRATING MASSES AND SPRING



$$M_1 = \left[\frac{KT^2}{4\pi^2 - KT^2/M_S} \right] \quad (11)$$

$$\frac{\Delta M_1}{M_1} = \left[\frac{M_S + M_1}{M_S} \right] \frac{\Delta K}{K} \quad (12)$$

$$\frac{\Delta M_1}{M_1} = 2 \left[\frac{M_S + M_1}{M_S} \right] \frac{\Delta T}{T} \quad (13)$$

$$\frac{\Delta M_1}{M_1} = - \left[\frac{M_1}{M_S} \right] \frac{\Delta M_S}{M_S} \quad (14)$$

FIGURE 3

the percentage of error in the mass determination, while in (13) the percentage of error in the period measurement is reflected almost doubly. But, as mentioned previously the T may be reduced by measuring time over many cycles. Also worthy of note in (14) is the insignificance of the station mass error.

It is uncertain that this vibrating mass system, when applied to man, will yield accurate enough measurements. This system is closely related to the first system where the spring accelerates the mass to a measured velocity; and for the same reason, energy absorption in the non-rigid body, it falls short. From a cursory look, the absorption appears to be more of a problem with this system, because during one cycle of T period two changes in the direction of acceleration occur. In other words, man's body, being the vibrating mass, acts as a damper which is not necessarily predictable, especially under prolonged weightlessness. Consequently, the accuracy of this system, with an unknown and possibly changing damper is exceedingly difficult to handle analythically.

CENTRIFUGE

Some of the larger space stations have been displaying internal centrifuges which are primarily for conditioning the astronaut with artificial g forces. It doesn't require much imagination to convert one of these centrifuges into a man weighing device. Substitution in the equation, $F = M\omega^2R$, for measured or otherwise determined values of F (centrifugal force), ω (angular velocity), and R (arm to center of mass), allows M to be calculated. The degree of exactness of such a calculation naturally depends on the accuracy of the individual inputs. There seems to be little question about precision measurements of the first two items F and ω; both of these can be measured fairly accurately with existing instruments. However, R, which for simplicity in this discussion is considered as not applying to the man and chair or couch but to the man only, is difficult to obtain with any degree of accuracy in an orbital laboratory. The combined effects of prolonged weightlessness (pooling of the blood, etc.) and the short term induced centrifugal force could cause the center of mass to shift from any previously determined location.

It also is doubtful that a centrifuge weighing system would be considered for small stations (10 ft. diameter or less) because of space limitations, a reduction in accuracy with reduced R (see (19)), and labyrinth problems (disorientation and motion sickness associated with the inner ear). It is known that astronauts, from both the U.S.S.R. and U.S.A. have experienced some difficulties with this problem; and that many people become nauseated when subjected to a few revolutions per minute on earth. There is much uncertainty about any threshold numbers pertaining to these physiological effects.

For the purpose of determining the possible accuracy of the centrifuge type scales for man in a space lab, sensitivity relations have been derived and are shown below in equation (19), along with reference equations and realistic assumptions of numbers for substitutions. The chart in Figure 4 shows the effects on the calculated mass, of errors in the measured inputs, for centrifuge g factors of 0.1, 0.5, and 1.

Reference Equations

$$F = M\omega^2 R = \frac{W}{g}\ \omega^2 R = \text{centrifugal force} \tag{15}$$

$$W = \frac{gF}{\omega^2 R} = \text{weight in lbs. } (g = 32.2 \text{ ft/sec}^2) \tag{16}$$

$$N = F/W = \frac{\omega^2 R}{g} = \frac{\text{centrifugal force}}{\text{weight}} = g \text{ forces} \tag{17}$$

$$\omega = \sqrt{\frac{Ng}{R}} = \text{angular velocity} \tag{18}$$

From the above chart it may be seen that errors in measurement of centrifugal force and angular velocity are not nearly as significant as a 1 inch error in cg location. It is also apparent that the former two decrease with an

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CENTRIFUGE ERROR ANALYSIS

INPUT ERROR	RESULTING ERROR					
	N = 0.1 g		N = 0.5 g		N = 1.0 g	
	%	LB.	%	LB.	%	LB.
ΔF = .18 LB. (0.1%)	1.01	1.80	22	.36	.11	.18
ΔR = 1 IN. OR .08 FT.	1.03	1.85	1.03	1.85	1.03	1.85
$\Delta\omega$ = .001 RAD./SEC.	.32	.57	.14	.26	.10	.18
TOTAL	2.36%	4.22 LB.	1.39%	2.47 LB.	1.24%	2.21 LB.

$$\frac{\Delta M}{M} = \frac{\Delta W}{W} = \frac{\Delta F}{F} \cdot \frac{\Delta R}{R} \cdot \frac{2\Delta\omega}{\omega} \quad \text{(SENSITIVITY EQUATION)} \qquad (19)$$

ASSUMPTIONS

W = 180 LB. MAN (COUCH OR CHAIR WEIGHT CONSIDERED 0)

R = 8 FT. ARM TO C.G. (CONSISTENT WITH 21 FOOT DIAMETER SPACE LAB)

FIGURE 4

increase in centrifugal g force while the latter appears to remain constant under any g forces. In reality, the center of mass would tend to shift outward as the body deformed more under increased g loadings. Even if this shift could be accounted for on earth, cardiovascular and other physiological effects over a long weightless period could make any conpensations invalid. The effect of R on W can only be reduced by increasing R which is generally limited by the vehicle diameter.

This centrifuge weighing system does not appear to have adequate accuracy for the 21 foot diameter space lab and even if this diameter is doubled and the R error halved, the accuracy still seems questionable for experimental laboratory work.

MOMENTUM METHOD

The fourth system makes use of the momentum concept which concentrates on the end conditions and by-passes the unknown energy dissipation occuring during the acceleration of a non-rigid body, namely man. The sequence of operation is as follows: A spring with a known amount of stored energy, E, reacts between the known mass of the space station, M_S and a small known mass, M_1, resulting in respective velocities of V_S and V_1. Values of these are obtained from a simultaneous solution of the following momentum and energy equations:

$$M_1 V_1 = M_S V_S \tag{20}$$

$$E = \frac{M_1 V_1^2}{2} + \frac{M_S V_S^2}{2} \tag{21}$$

resulting in:

$$V_S = \sqrt{\frac{2 M_1 E}{M_S (M_S + M_1)}} \tag{22}$$

and

$$V_1 = \left(\frac{M_S}{M_1}\right) \sqrt{\frac{2 M_1 E}{M_S (M_S + M_1)}} \tag{23}$$

To prevent rotation of the space station and thus to keep equation (21) valid, the line of action is designed to be through or very near its c.g. Next M_1 contacts the man and supporting carriage, M_2, and all move as a unit with a velocity of V_3 which is relative to the original velocity of the space station et al. By conversation of momentum:

$$M_1 V_1 = (M_1 + M_2) V_3 \tag{24}$$

and

$$M_2 = \frac{M_1 (V_1 - V_3)}{V_3}$$

Since the station, as a result of its first reaction, is moving slowly in the opposite direction, a measured velocity V_M between the station and carriage is the sum of V_S and V_3 or:

$$V_3 = V_M - V_S \tag{25}$$

Substituting (25) into (24) gives:

$$M_2 = \frac{M_1 (V_1 - V_M + V_S)}{V_M - V_S} \tag{26}$$

When V_M is measured, the relative position of all parts of the body is unimportant as long as all parts are moving as a unit. This seems to occur externally a very short time after the accelerating force is removed (displace the skin or a muscle and see how quickly the oscillations die out). There seem to be good reasons to believe that internally the response is similar. Therefore after a short period of time, a second or so, time measuring devices measure t over a known distance s. From $V_M = s/t$, V_M can be determined. Substitutions into equation (26) will now yield the mass of the man and carriage.

Next it becomes advisable to determine how accurate this device really is. Therefore equations, reflecting the sensitivity of M_2 to input errors were developed by differentiation and are shown below:

$$\Delta M_2 = \left[\frac{V_1 - V_M + V_S}{V_M - V_S}\right] \Delta M_1 \tag{27}$$

$$\Delta M_2 = \left[\frac{V_S (V_1 - V_M + V_S)}{V_1 (V_M - V_S)}\right] \Delta M_S \tag{28}$$

$$\Delta M_2 = \left[\frac{M_S V_M V_S}{2E}\right] \Delta E \tag{29}$$

$$\Delta M_2 = -\left[\frac{M_1 V_1 t}{(s - V_S t)^2}\right] \Delta s \tag{30}$$

$$\Delta M_2 = \left[\frac{M_1 V_1 s}{(s - V_S t)^2}\right] \Delta t \tag{31}$$

Other degradations to the accuracy of this system exist at the guide rollers in the forms of inertia and friction. However, the inertia becomes insignificant if the rollers are small in diameter and are made from some light plastic material. The frictional losses are investigated in the following exercise, which includes Figure 5.

e = eccentricity of cg to line of force

$\int F dt$ = impulse from M_1 impacting M_2

$\int F_1 \, dt$ = resulting impulse at rollers

f = equivalent horizontal coefficient of fraction at roller contact

Taking moments about cg:

$$e \int F \, dt = L \int F_1 \, dt \tag{32}$$

$$2f \int F_1 \, dt = 2 \frac{fe}{L} \int F \, dt = \text{total horizontal frictional impulse} \tag{33}$$

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MOMENTUM METHOD

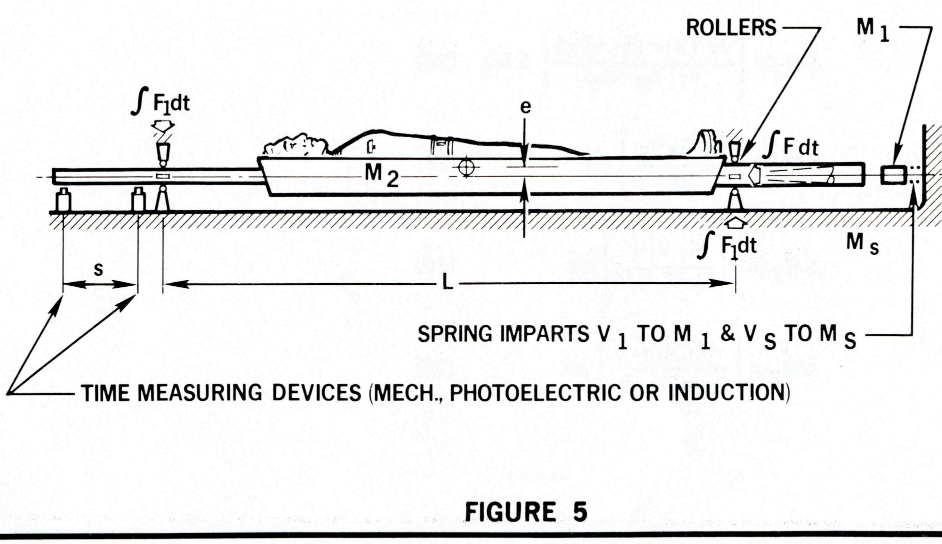

FIGURE 5

but since $M_1 V_1 = \int F\, dt$

$$2 \frac{fe\ M_1\ V_1}{L} = \text{horizontal momentum exchange to frictional impulse.} \qquad (34)$$

$$\Delta V_3 = \frac{\text{Momentum Change}}{M_1 + M_2} = \frac{2M_1\ V_1\ ef}{(M_1 + M_2)\ L} = \qquad (35)$$

velocity reduction caused by friction.

From equation (24) the following sensitivity equation can be obtained:

$$\Delta M_2 = - \left(\frac{M_1 + M_2}{V_3}\right) \Delta V_3 \qquad (36)$$

This may be reduced by substitutions to:

$$\Delta M_2 = - \frac{2(M_1 + M_2)(e)(f)}{L} = \text{effects of man's c.g. location and roller friction} \qquad (37)$$

A summation of all sensitivity terms give the maximum possible error for a given set of assumptions. Finally to apply this error analysis to a conceivable system, the following assumptions (not necessarily optimum) and results are shown below:

M_1 = 2/32.2 = .062 slugs (2 lb. weight)

M_2 = 180/32.2 = 5.58 slugs (180 lbs. includes man and carriage)

M_S = 25,000/32.2 = 776.4 slugs

E = 1 ft lb of energy in the spring

$$V_S = \sqrt{\frac{2\ M_1\ E}{M_S(M_S + M_1)}} = .000454 \text{ ft/sec} \qquad (22)$$

$$V_1 = \left(\frac{M_S}{M_1}\right) V_S = 5.675 \text{ ft/sec} \qquad (23)$$

$$V_M = \frac{M_1\ V_1 + M_2\ V_S + M_1\ V_S}{M_1 + M_2} = .063 \text{ ft/sec} \qquad (26)$$

s = 1.000 ft

t = 15.900 sec.

L = 8 ft

e = .08 ft (1 inch eccentricity of c.g.)

f = .007 equivalent coefficient of horizontal friction at the roller guide, transferred from the roller ball bearing (.015)

ΔM_1 = .000068 slugs (1 gram by weight)

ΔM_S = .776 slugs (0.1%)

ΔE = .015 ft lbs

Δs = -.0008 ft (.010 inches)

Δt = .005 secs. (.001 is feasible)

With reasonable assumptions the momentum system shown has a maximum possible error of approximately .37%, which should be within the accuracy required. See Figure 6. Since all errors with the exception of friction are plus or minus, the probable error would be even less. In the above example, deviations in s and M_1 are rather effective on the final results. Both of these along with E can be measured quite accurately before the actual flight. Another point of interest is the small influence of friction, which is affected by the man's cg location. In fact his cg location affects friction only. Thus, it now appears feasible to retain the man in a sliding chair rather than a sliding couch and take the chance of the cg being off more. The chair would reduce the length of the mechanism. See Figure 7.

The momentum scales appear to be better adapted for keeping a weight check on man in space than the other versions studied, because of the following characteristics:

(a) A high degree of accuracy.
(b) Insensitivity to man's c.g. location or shift.
(c) By-passing of energy dissipation, an unknown loss occuring in the acceleration of man.
(d) Minimizing of motion sickness.

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MOMENTUM METHOD ERROR ANALYSIS

TYPE OF ERROR	ΔM_2 (SLUGS)	ΔM_2(LBS)	% OF M_2
t	.0018	.058	.032
s	.0045	.145	.081
E	.0016	.052	.029
M_1	.0062	.200	.111
M_S	.0057	.185	.102
FRICTION	.0008	.025	.014
POSSIBLE TOTAL ERROR	.0206	.665	.369

FIGURE 6

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MOMENTUM METHOD CHAIR ARRANGEMENT

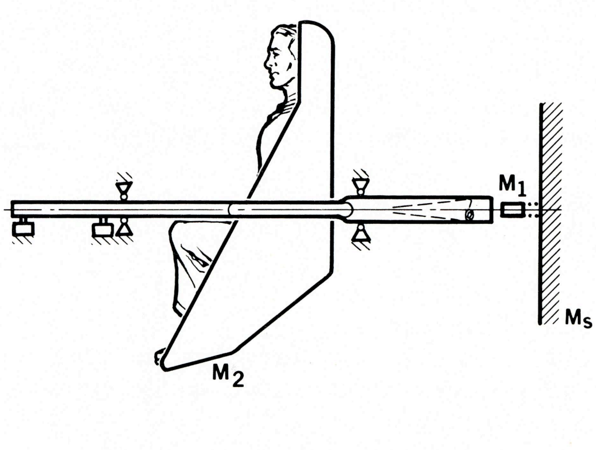

FIGURE 7

CONCLUSION

Of the four mass determining methods studied, none presented any specific problems for rigid masses. Therefore, the concentration of effort has been on weighing the non-rigid mass, an operation which can possess problems. For keeping an accurate weight check on man (a non-rigid mass) under prolonged laboratory conditions at zero-g, all but the last method, the momentum method, were found to have possible disqualifying deficiencies. Indeterminate energy absorption in the non-rigid body was shown to degrade the accuracies of both the energy-velocity and vibrating mass and spring methods; however, the degree of degradation cannot be determined analytically. The shortcomings found in the centrifuge method were: (1) an indeterminant center of mass location; (2) labyrinth problems; (3) small laboratory limitations. Most of the problems associated with the first three methods are of an unknown or indeterminate nature, qualities which are undesirable for laboratory measurements. The fourth method, called the momentum method, is all but free of the disadvantages noted in the other systems. It is unaffected by energy losses. The unknown cg location of man, which can contribute to frictional losses only, has a very small effect on the accuracy of the system. In fact, this effect is so small that consideration could be given to a chair arrangement where the location of the cg might fall over a wider range, instead of to the couch system shown in figure 5. A chair support would reduce the overall length of the system. Since there is no rotary motion with the momentum system and rectilinear motion is small, motion sickness would not be aggravated. One unknown is the time it takes for the body to settle down and act as one mass after it has been accelerated. This does not appear to be a problem, though, because of the damping quality of flesh. Extension of the settling time could alleviate any possible problem of this nature. Neither is circulation of the blood expected to affect this system because the net momentum in a closed system can only be zero. The respiratory system not being a closed system, could provide an unwanted impulse, depending on the direction of expulsion. This possible problem can be easily eliminated by having the subject hold his breath.

From a random example with realistic tolerances, it was found that the total maximum error for the selected momentum system was approximately 0.38%. This number, although probably acceptable, could no doubt be improved by optimizing the inputs and tightening the tolerances.

This study, with a preliminary look, has indicated that scales, for weighing man in a zero-g orbit with assurance of accuracy, could be designed and built on the momentum principle, whereas devices based on the other methods may lack the desired precision. However, it would be advisable to further pursue studies on all these methods, especially as pertains to accuracies, optimization, station cg locations, degradations, and simulation tests. In the final analysis a certain amount of testing will be necessary, mainly because of that unknown quantity, the human body.