42

1

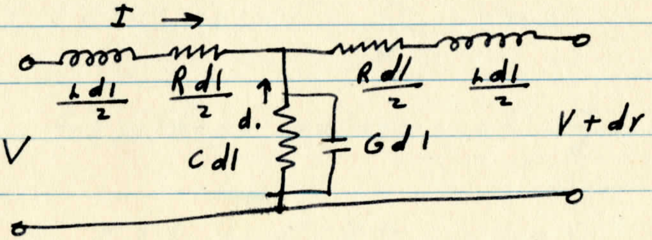
Antennas

&

Transmission Lines

Ear Piece

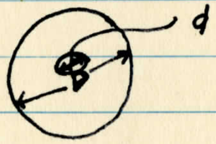
equiv. circuit for
<u>dl</u> of transmission line
G = shunt conductance



characteristic impedance $= \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$

$$L = \frac{1}{C\ (3 \times 10^8)^2}$$

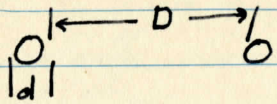
for coaxial line



$L = \frac{K_m \mu_0}{2\pi} \ln \frac{D}{d}$ henrys meter

$C = \frac{2\pi K_e \epsilon_0}{\ln \frac{D}{d}}$ farads per m.

with air dielectric K_e = dielectric const of material
K_m = permeability " " "
$Z_0 = 130 \log \frac{D}{d}$ ohms ϵ_0 = diel. const. of free space
μ_0 = perm. " " " "



$$Z_0 = 120 \cosh^{-1} \frac{D}{d} \text{ ohms}$$

for large values of $\frac{D}{d}$ this becomes

$$Z_0 = 276 \log \frac{2D}{d} \text{ ohms}$$

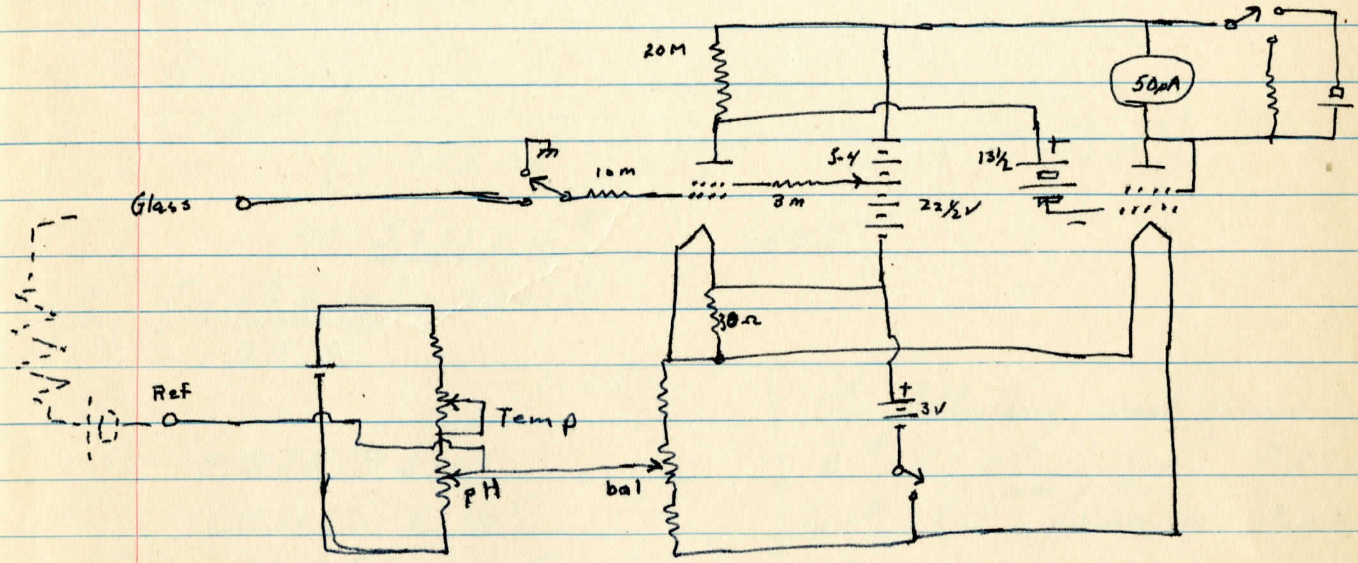
for use with dielectric

$$Z_0 (\text{dielectric}) = \frac{1}{\sqrt{K_e}} Z_0 (\text{air})$$

K_e = dielectric const of dielectric

Commercially available meters seem to leave much to be desired as regards to stability, simplicity of operation and cost.

The Beckman G.S. is a good example of such short comings -



simplified

From G.S. manual

$$pH_{sample} = pH_{buffer} + \frac{E_b - E_s}{\frac{2.3026\,RT}{F} \times 10^3} = pH_b + \frac{E_b - E_s}{.19842\,T}$$

R = gas const. 8.3144 j/°m

F = Faraday 96496 coul/eq

T = °Kelvin

E_{std} in millivolts

$$= pH_b + \frac{E_b - E_s}{59.15} \quad T = 25°C$$

This meter use KCl buffer in electrode - electrode out put ≈ 60mv/pH unit + 50 to 10^3 Meg-Ω

Statham calibration calculations

Ave- of existing transducers-

Cal. F. F - 52.77 μV/Volt/cm Hg

R - 297.3 Ω

$$R_c = \left(\frac{10^6}{4NF} - .5\right) R$$

Cm Hg

N	R_c
2.5	$5.71 \times 10^5 = \left(\frac{10^6}{4 \cdot 2.5 \times 52} - .5\right) 2.97 \times 10^2 = \left(\frac{10^6}{10^1 \times .52 \times 10^2} - .5\right) = 1.923 \times 10^3 \times 2.97 \times 10^2$
	$\approx 5.711 \times 10^5$ 1895
2.5 / 5.	5.652×10^5 exact / .9478 $\frac{1}{1055}$
5.	2.82579×10^5
10	1.41155×10^5
15	$.94053 \times 10^5$
20	.70488
25	$.563787 \times 10^5$
30	$.469524 \times 10^5$

The following was meas. c̄ 56.0K resistor - P23d

Ser #	Cal	mm defl. 2.5 mm/Hg	% error
446	20.5	22.5	
1236	19.5	18.0	
210	20.	20.3	

N calc. for #210 where R = 297 F = 52.07

$$N = \frac{10^6}{4F}\left(\frac{R}{R_c + .5R}\right) \quad R_c = 56.37 \times 10^3$$

2.524

$\frac{.024}{2.5} \approx \frac{2.5 \times 10^{-2}}{2.5} = 10^{-2} \times 10^2 = 1\%$

N. calc. for #78 where R = 307.9 F = 50.97

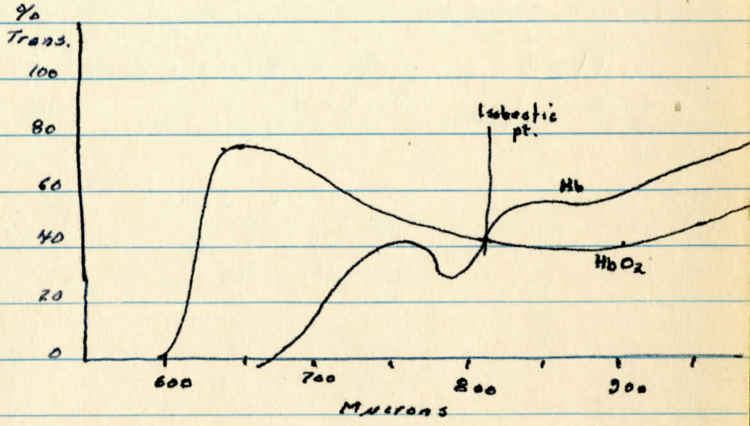
$R_c = 56.37 \times 10^3$

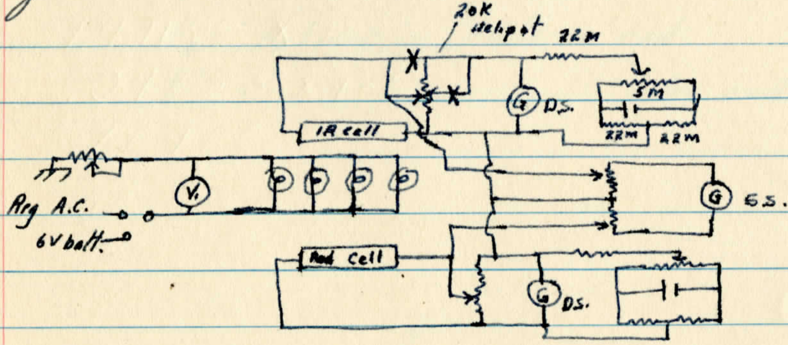
$= 2.671$ $\frac{.17}{2.5} \approx 6.8\%$

Oximetry

Transmission of blood is measured by photocells at wavelengths which allow determination of total Hb and HbO_2. At one time green ($\approx 5 \times 10^3$ A°)[1] was used to determine Hb + HbO_2 and red ($\approx 6.5 \times 10^3$ A°)[1] HbO_2. Later investigators use 8×10^3 A° to determine Hb + HbO_2 since their transmission is equal at this point (isobestic pt.).

One of the better available instruments is the Waters X 70A Oximeter. This device is straightforward using generating cells to drive a dual and/or single galvonometer.





A multiple light source of 6(?) Mazda #51 bulbs operated @ 3.5 volts, monitored by a voltmeter. Two selenium iron cells broadly sensitive at 6.50×10^3 A° + 8×10^3 A° without filters are used to determine transmissivity at these points. Determinations are made either by the double scale D.S. or single scale S.S. galvos.

The instrument seems satisfactory but its bulk and

[1] Eastman wratten 25 A and 85 filters

mechanical sensitivity preclude its use in O.R., etc. Operating instructions were good and contained an excellent bibliography.

A second effort was found in July 58 PGME in which A.C. amplification was attempted by modulation of a G.E. 1448 bulb at 20 ~, 30% duty cycle which supposedly resulted in 100% modulation. Filters and sel. cells were used to drive a 3 stage pentode amplifier and a redundant 'computer' to drive an Est. Angus - Drift in photocells & unreliability were problems - Good bibliography.

$$\% \text{ sat} = \frac{E_{Hb}}{E_{HbO_2} - E_{Hb}} - \frac{E'_{Hb}}{E_{HbO_2} - E_{Hb}} \cdot \frac{\log R - \log R_o}{\log IR - \log IR_o}$$

R, IR = blood transmission

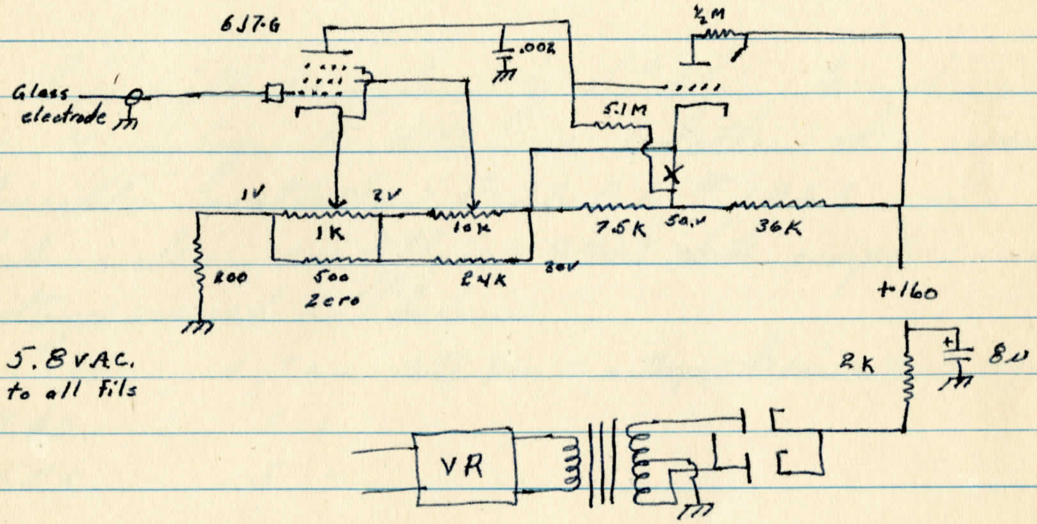
R_o, IR_o = bloodless

E = extinction coeff. in red

E' = " " " infra red

12-31-59 **pH meter**

A Cambridge Research Model pH meter was examined. Electronics were crude, consisting of; a 6H6 full wave rectifier supplied by a V.R. trans ~~with~~ giving a D.C. output of +160 V. filtered by a single 8μ and 2K filter - A 6J7G drove a 6E5 indicator. A conventional potentiometer arrangement was used in which a std. cell was used to calibrate the voltage from dry cells. Temp. controls were standard and the cell was Ag - Ag-Cl, HCl ‖(Glass) unknown | KCl_{sat} | HgCl | Hg -



Cardiac Work

① Energy = Pressure × Volume + ½ Mass × Velocity^2

② $Work = P \cdot \frac{dV}{dt} + \frac{1}{2} M \left(\frac{dx}{dt}\right)^2$

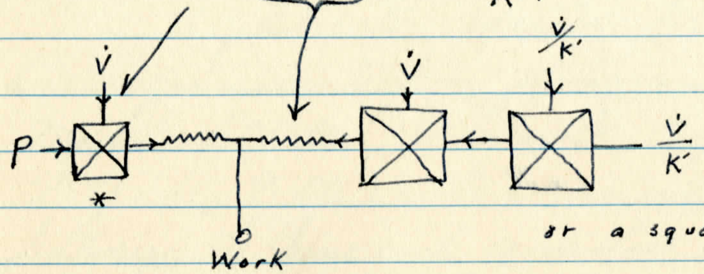
$= P \cdot \dot{V} + \overbrace{K}^{\frac{1}{2}K} \cdot \dot{V} \left(\frac{\dot{V}}{K'}\right)^2$

P - meas. (Statham Transd.

$\frac{dV}{dt} = \dot{V}$ - meas. (Flow meter

$M = Density(K) \times \dot{V}$

$\frac{dx}{dt} = \dot{V} / Area\ (of\ vessel\ lumen)\ K'$



or a squaring ckt.

* This might be replaced by Statham transducer excited by $\dot{V}$ analog - to determine whether this was feasible a Statham P-23-D6 #1263 was balanced in an Offner 9803 S.G. coupler and voltage varied as below -

1.52V	<.5 mm shift from 0 - Max Gain - ≈ 10 required
3.10	1 "
4.65	5. "
6.10	12. "

A transient occurred at increased voltages (>3.) which caused an exponential shift away from 0. This is likely a temp. effect which should cause no trouble at reduced voltages. Varying input voltages caused the expected directly related varying shifts -

1-4-60

In Dec. '59 (?) Review of Sci. Instruments, Oak Ridge reported on measurement of grid current of receiving tubes. This relation predicts their behavior to a factor of 2: $I_{grid} = E_{grid-plate}^{3/2} \, I_{kathode} \times 10^{-10}$ Amp
Some values were 6AN5 at 20V-, 10^{-3} I, Gain=9 $I_g \approx 10^{-10}$
Low filament voltage is essential.

A photometer adapted for flourescent dye circulatory time measurements from photogrammetric use (Photovolt Model 501-M, Photovolt Corp., 95 Madison Ave. N.Y. 16, N.Y.) and a recorder had some interesting features. The basic unit consisted of a balanced direct coupled amplifier for a phototube (6J5's + triode 6K6's) which measured flourescence of a greend dye (Flourescin) which was excited by a blue filtered incandescent bulb (auto lamp). Most interesting feature of the recorder was a photo-conductive chopper driven by

Magnitude of voltage produced by thermal agitation

$$E^2 = 4KT\int_{f_1}^{f_2} R\,df$$

K = Boltzman's Const. = 1.374×10^{-23} joule/°K

T = Abs. Temp.

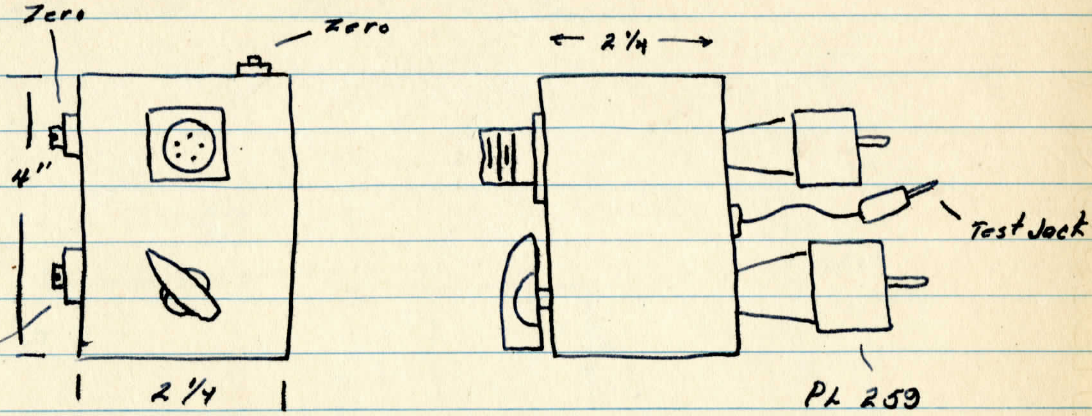
R = Resistance component of impeadance producing voltage f (freq)

f = freq

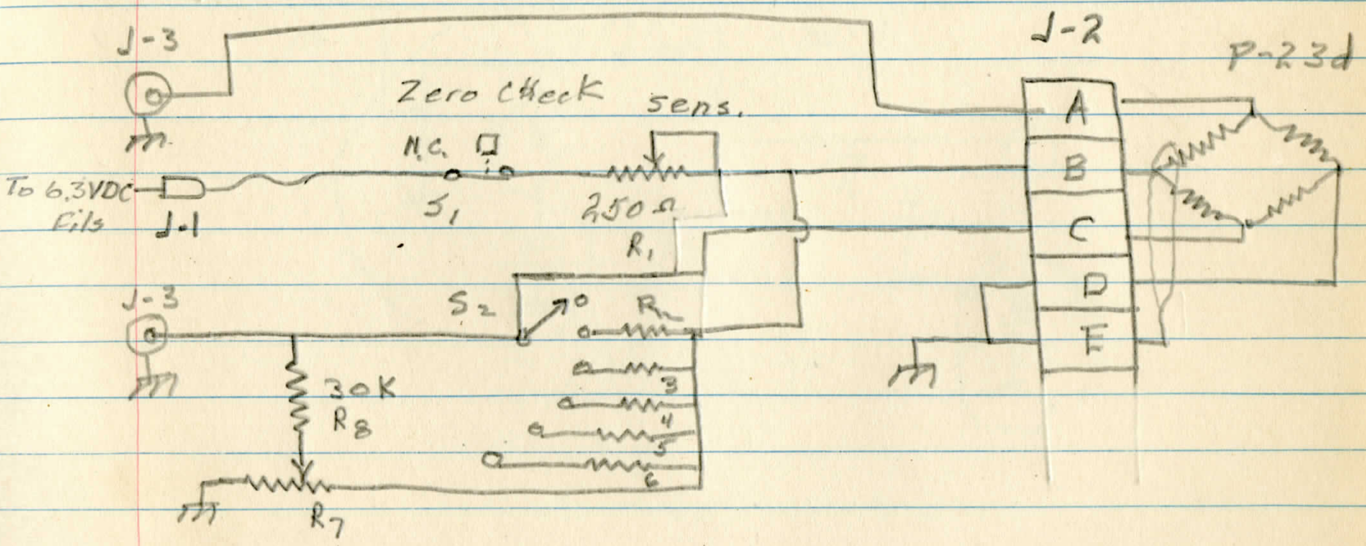
For const. resistance component

$$E^2 = 4KTR(f_2 - f_1)$$

5-7 Coupler for Statham P-23D to Tek 503 Scope. P-12



Bud MiniBox



R7 - 2500 Ω AB CLU 2531

R8 - 30K WW

R1 - AB CLU 2511

J-1 Test Jack

R2 -

J-2 Cannon MS-3102A 14S 5SC

3 -

J-3 PL-259

4 -

5 -

S-1 - SPST N.C. Grayhill 4002 orcq.

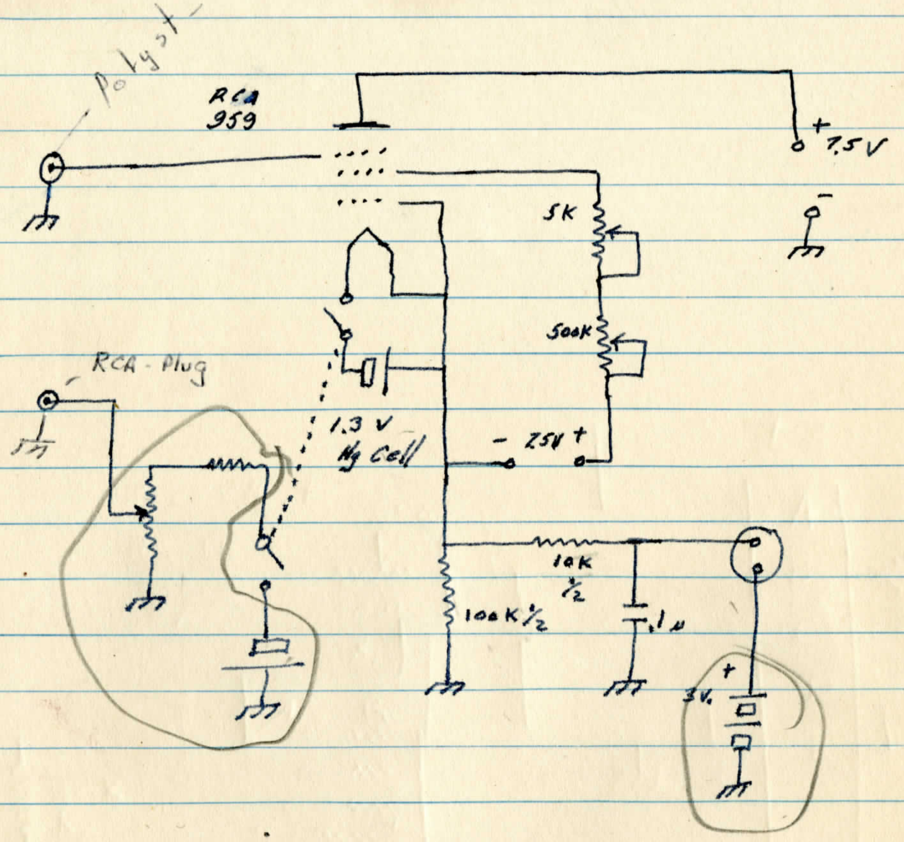
6 -

S-2 SP6T - Grayhill 5001-6

7 -

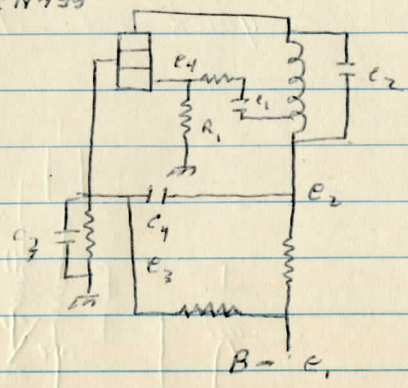
1.50

Electrometer Cathode Follower



MKI Telemetry ser #2

2N499



L_1 - 6 T # ¼" I.D. 3/8" long

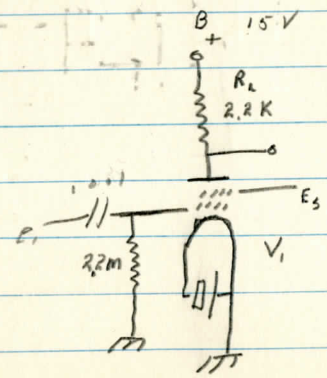
R_1	300 Ω	1/10 W	R_5	1500 Ω	1/10 W
R_2	200 Ω	1/4 W	$C_{1,2}$	12 μμf	
R_3	2100 Ω	1/4 W	$C_{3,4}$	.001 μ	
R_4	12 K	1/10 W			

f_{osc} = 98 MC

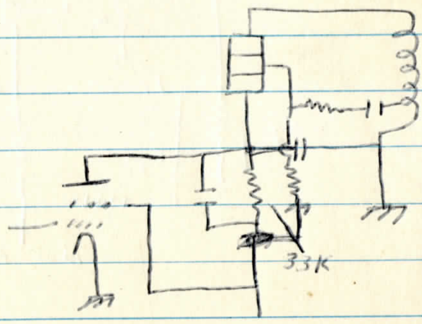
drop across 1500 Ω =

e_1	e_2	e_3	e_4
-17.5	-9.0	-2.0	-1.8, N .54

Attempt to modulate preceding Osc. c̄ V.T.



quick opt. of ckt.

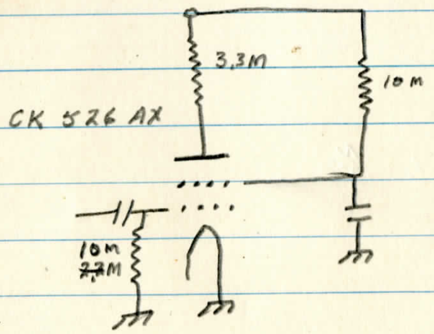


V_1 = CK 526AX

170 mv. applied to e_1 drove the osc. nicely - ≈ 50 mv. should be sufficient - ckt. seemed stable

From this a gain of 50 X should be obtained for Amp. c̄ above & r_L = 3.3 M r_g = 10 m byp. c̄ .05 & $\frac{e_o}{e_{in}} = \frac{1.5V}{6 \times 10^{-2}} = 25$

by juggling the opt. ckt. below was evolved

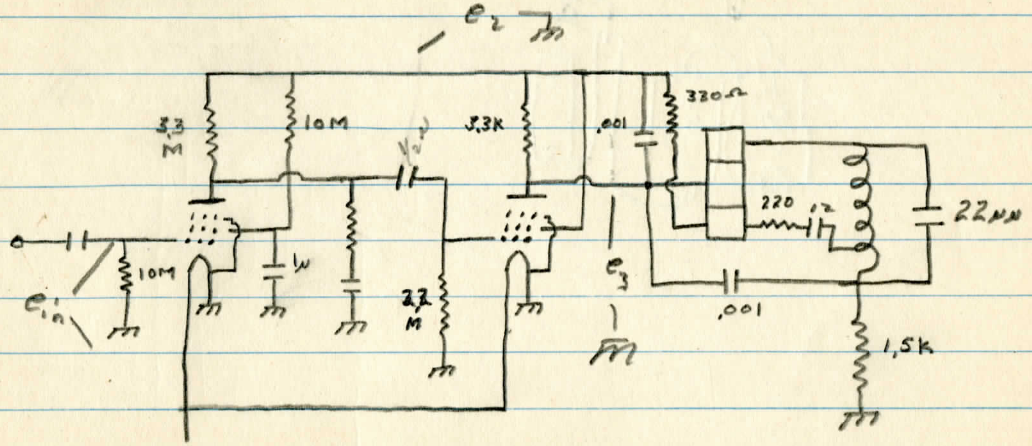


15

22

$10^{-6} \times 10^{2} = 10^{-4}$

MK II Telemetry



Parts- Min switches
Batteries - 15V
Resistors
Capacitors -

e_{in}	e_2	e_3
2×10^{-3}	4×10^{-2}	10×10^{-3}

Problem: Single reliabiable optical earpiece densitometer to measure arterial blood saturation, cardiac output from injected dyes, and possibly B.P. $\bar{c}$ auxillary pressure devices. A pulse output will be coincidentally present.

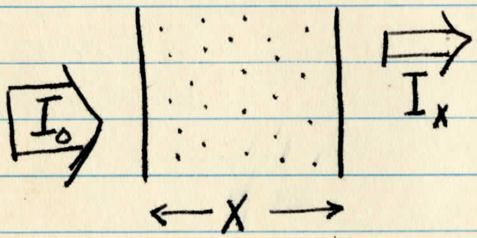
Theory: $K = k C$ (Beer's law)

deviations are: non-parallelism
inhomogeneity
reflection from surfaces in path
~~and~~

k = coeff. / unit conc.
C = Concentration

$$\text{Log } I_0 / I_X = k C X$$

I_0 = incident rad.
I_X = transm. "
X = path length



$$\text{Density} = \text{Log } I_0 / I_X = -\text{Log } I_X / I_0 = -\text{Log } T$$

$$\text{Conc.} = -1/K \text{ Log } I_X / I_0$$

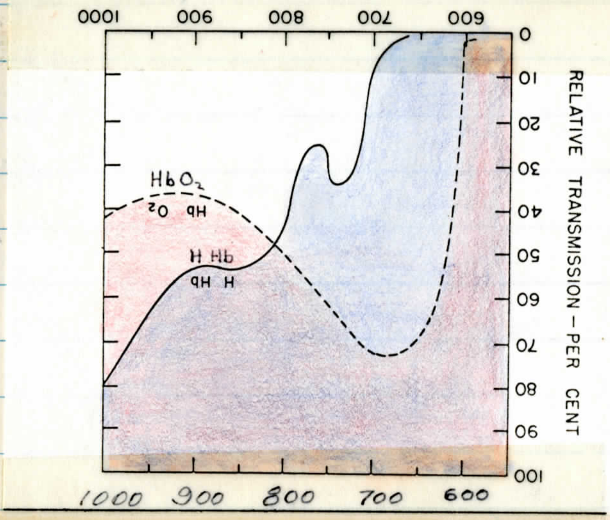


Fig. 2

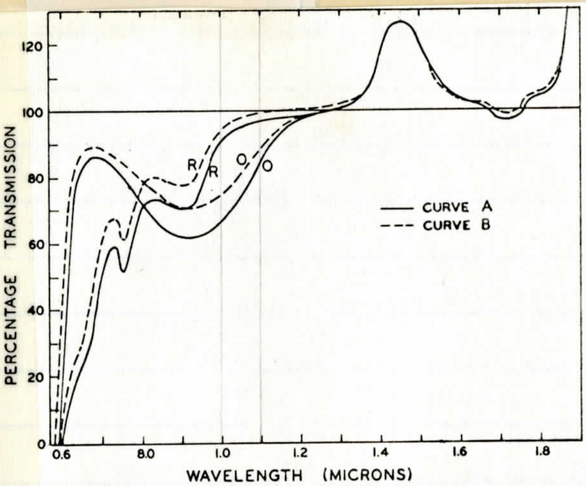


Fig. 1. Percentage transmission curves relative to distilled water for oxygenated (*O*) and reduced (*R*) hemoglobin with Curves A and B representing hemoglobin concentrations of 11.65 gm./100 cc. and 8.45 gm./100 cc., respectively. Optical path, 0.1 cm.

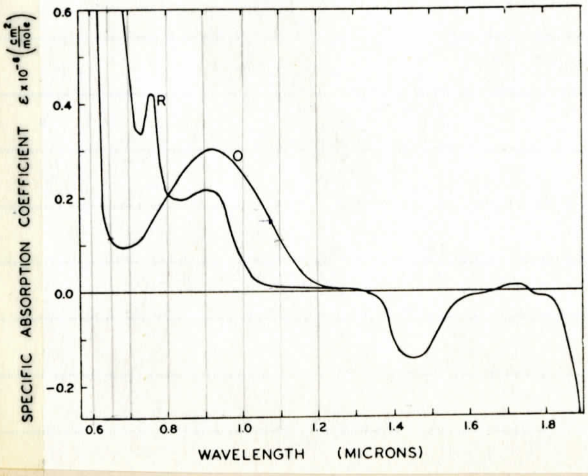


Fig. 2. Average extinction coefficients of oxygenated (*O*) and reduced (*R*), as obtained from three pairs of runs, of which two are shown on Fig. 1

The spectral transm. of reduced + oxygenated hemoglobin allows measurement of these substances* by densitometric methods - The practical instruments have been made in two general configurations but principles are the same. By standardized measurements at 805 mμ the total quantity of blood is measured.

*conc.

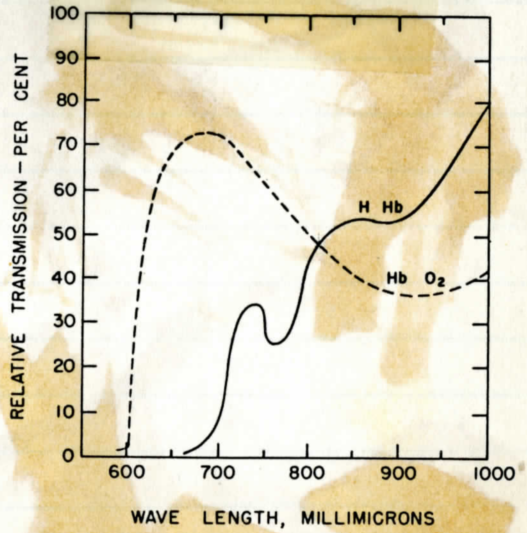


FIG. 2.—Comparison of spectral transmission of oxyhemoglobin (HbO_2) and reduced hemoglobin (HHb). Note marked difference in transmission of incident light of 640 mμ and equal transmission of light of 800 mμ by HbO_2 and HHb.

At 805 mμ the density of Hbb + HbO_2 are equal and thus have equal effects on transmission of light + can accurately be assumed to be the same substance.

At say 650 mμ the ~~dens~~ rel. absorption of light by HHb is relatively great while that of HbO_2 is much less. This is more easily appreciated if the above curve is inverted as in fig 2. A device which measures the light output at this point will have its output: 1) increased by ↓ amounts of HbO_2, 2) ↓ by increased amounts of HbO_2 3) ↓ by ↑ total blood quantity

3) is measured as previously described at the 805 mμ spot. 1) + 2) are inversely related such that a curve of the

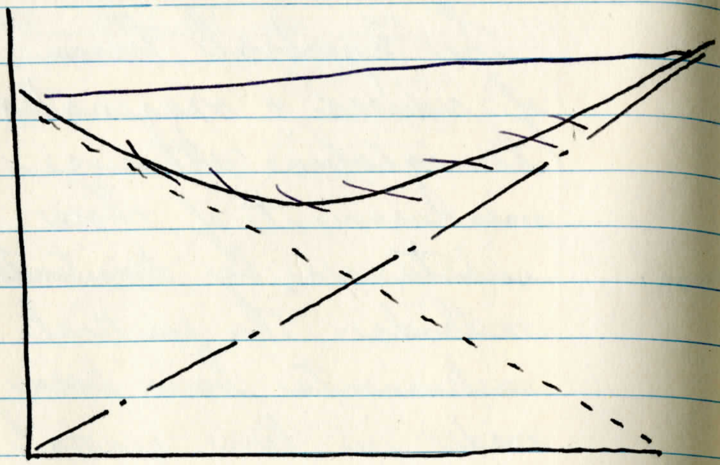
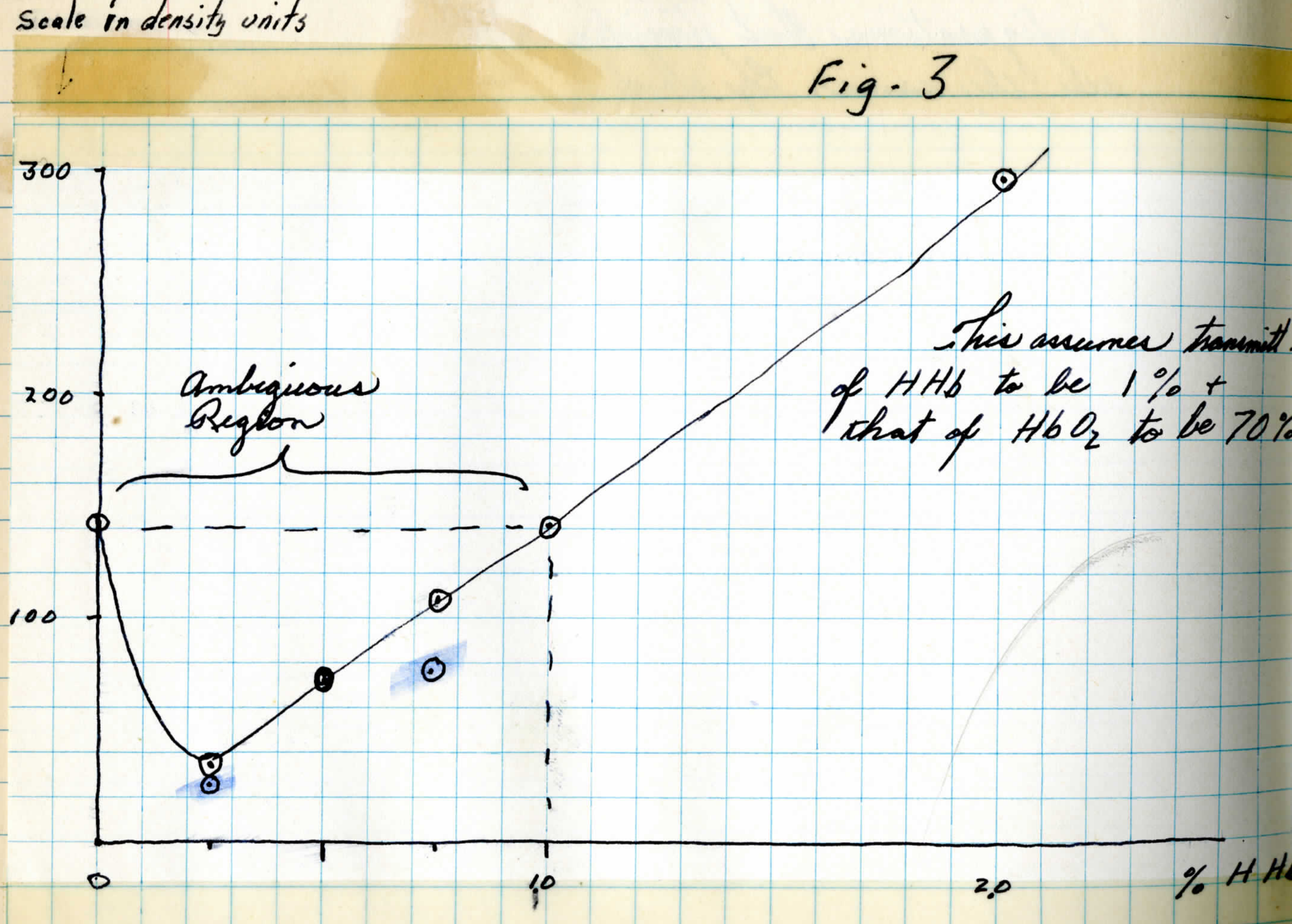


Fig. 3

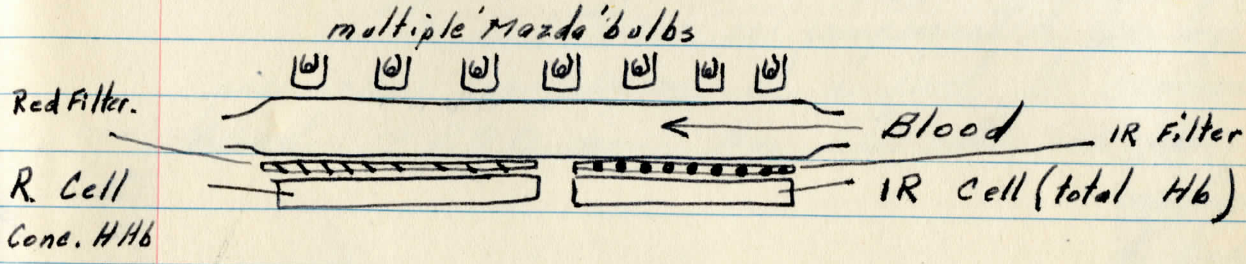


Scale in density units
300
200
100
0
10
20
% HHb
Ambiguous Region
This assumes transmitt[ance]
of HHb to be 1% +
that of HbO_2 to be 70%

Fig. 4

general shape of fig. 3 must occur. If the difference in relative transmission is great as it is in this case this "crossover" will occur at low concentrations of HHb. See Fig #4. Except for small discrepancy and after correction for total blood vol. (red cell mass) the ~~output of this~~ measurement of intensity at 650 mμ will then be ~~the~~ a funct. of the inverse anti-log of reduced Hb (HHb).

One general configuration of an oximeter is the cuvette shown below which is used for in vivo measurements.



Corrections or rather errors may be present if foreign substances such as bile or other pigments are present. Inherent errors are in the calibration method which is empiracally done against Van Slyke or other methods and may also accrue from cell nonlinearity, temp. drifts, changes in light output and system non-linearities. Filters previously used have been broad rather than narrow types. This leads to errors for the curves are by no means symmetrical of either side of the

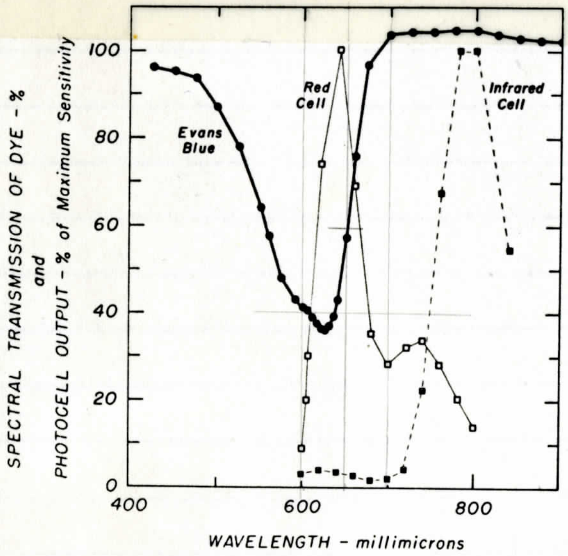
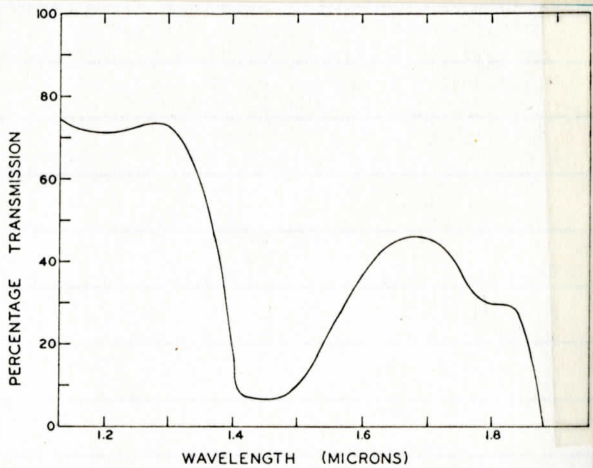


FIG. 3.—Comparison of light transmission of Evans blue (5 mg/l) in plasma and spectral sensitivity of the oximeter photocells. Note that Evans blue has its peak absorption at approximately the same wavelength for which the oximeter red cell has its peak sensitivity and that it absorbs practically no light in the region of 800 mμ, which is the zone of peak sensitivity of the infra-red photocell.

Fig. 3. Infrared transmission of distilled water relative to air. Optical path, 0.1 cm.



isobestic point + even if this were the case they would be accurate for only one relative concentration of HbO_2. The response curves demonstrated in fig for a Woods Oximeter will allow appreciable response to HbO_2 conc. + push the "crossover point" into lower concentrations of HbO_2.

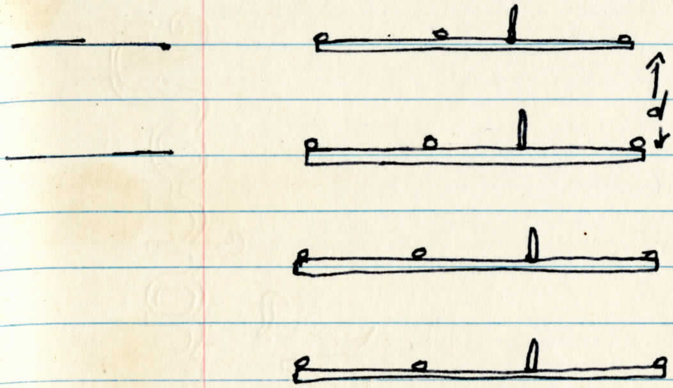
Earpiece oximeters have been devised which work well and it ~~it~~ is this configuration that we desire. A number of mechanical problems are introduced here as well as an additional density in the optical path, the bloodless ear tissue.

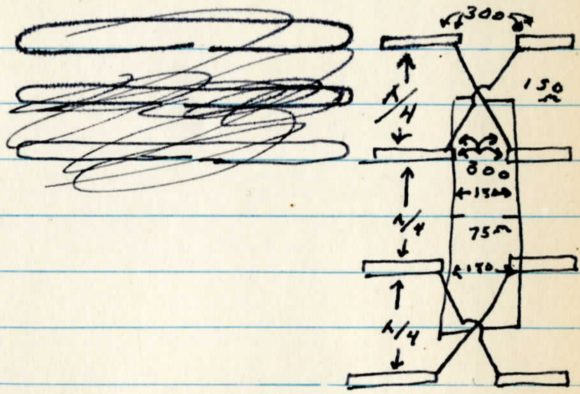
The ear tissue is compensated for by pneumatic compression at a level to remove blood, ~ 200 mm. Hg. The cells are then balanced at this point and their sensitivities adjusted to compensate for the reduced light reaching them. The tissue is reported to be essentially opaque below 600 mμ, but the exact transmission curve and its lower freq. cut off are not reported + need not be considered so long as they are constant.

Another problem is the presence of venous blood in tissue. This is apparently neglegible after heating to "arterialize" (increase blood flow) the ear. At least it has been neglected in all published work thus far. The heating is supplied

1 - Wood, E.H. - Oximetry - Med. Physics VII P665

by the light source (a 6 V. .25 A - 1.50 w bulb) in the Woods ear piece. See the opposite page - As currently used





set d initially at λ/4 for two stack + phase properly - feeder is 300 Ω tapped at midpoint with 150 Ω

$$\left.\begin{array}{l}\text{Maximum possible dist. for}\\ \text{direct-ray transmission}\end{array}\right\} = 1.225\,K\left(\sqrt{h_s} + \sqrt{h_r}\right)$$

$$K = \frac{\text{effective earth radius}}{\text{actual " "}}$$

= usually 4/3 = 1.33

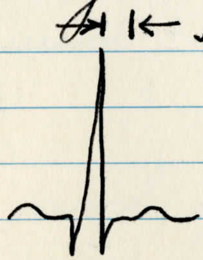
h in feet

D " miles

Prob.: to transmit period of pulses-
The pulses will be derived from the EKG QRS complexes and basic considerations of resolution are as follows-

1. ~~basic~~ minimum "bit" information in EKG- 100 samples at 5 bits could be considered minimum - or 5 K B/S

2. Time resolution of sampler of QRS Probably will not exceed 5×10^{-3} S



3. Max. and minimum heart rates: Personally feel practical limits are <u>30</u> & <u>250</u> B.P.M. or T = 2. sec & .24 sec.

4. Desired accuracy - would chose .02 sec

5. Max. rate of change of rate - these could be large & ~ entire cycle or say

A 1.8 c/sec to .24 c/sec -

Col. Ord feels 40 to 200 BPM c̄ max. rate of change to be 40 to 200. Do not consider arrythmia's outside of this rate.

The following assumptions were made on the processes involved -

A: That binary code would be used

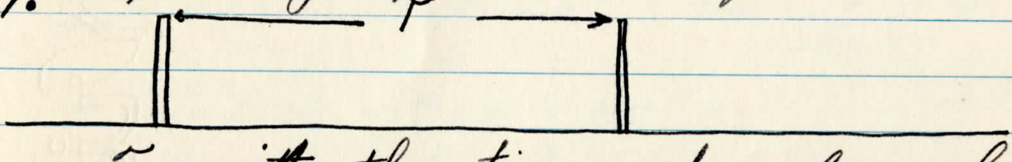
B - That channel width remains fixed

Definition: B/S ≡ bits/second of digital analog data
B ≡ bits " " "
bits/second ≡ bits/second of data in binary code
bits ≡ bits " " " " "

Since much of the data will not involve large rates of change ie the rate will be relatively fixed or vary slowly except for arrythmias the total bit rate will be small however the bandwith must remain large unless some form of storage involving only total bits is resorted to -

The following schemes are possible:

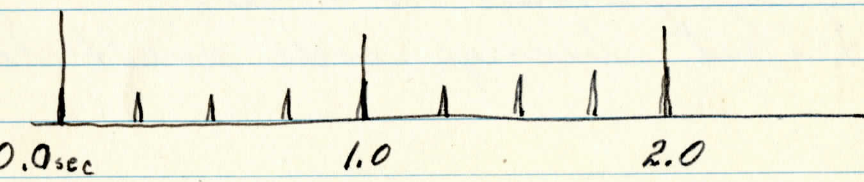
1.



Transmitt the time value of each pulse interval from zero. ~~With a max. rate of 250 rate BPM & prev. assumed value of .02 sec. this would be~~ This

20 Jan 64

At max. rate of 250 BPM.



$\frac{.24}{.02} = 12 \times 4$

in binary 5 bits - & in 1 sec - 20 bits/sec

& lowest rate: 2 sec ∴ $2 \times 50 = 100$

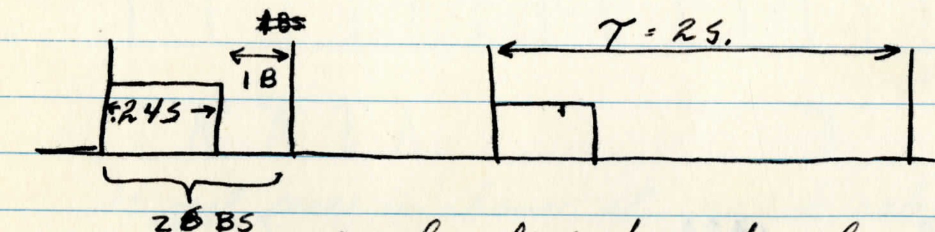
& in binary 8 bits total or 4 bits/sec

B/S = Bit/sec (This is tot. bits. of info)

B/M = beats/min

$\underline{\tau} = \tau -$ dead space

method essentially uses the max. bandwidth or 50 B/s. ~~Some form of storage of the total amount of data~~. Some improvement in reduction of data can be effected by introduction of a blank space corresponding to or slightly less than ~~than~~ τ of the highest rate to be counted.



At the highest rate this could afford a great saving of ~ ≈ 48 B/S neglecting coding of the dead space. A "typical" rate of 60 B/M would now require only $\frac{1.00 - .24}{.02} = \frac{.76}{.02} = 38$ B/S and the lowest rate would have a saving of only $\frac{.24}{2 \times .02} = \frac{.24}{.04} = 12$ for a B/S of $50 - 12 = \underline{38}$.

In terms of binary coding: with the above "dead space" the bits/sec are only 4 at max. rate: at 120 BPM ∴ $\tau = .50$ &

$\underline{\tau} = .50 - .24 = .16$ $B = \frac{.16}{.02} = 8$

& bits/sec. = ~~3~~ 4 × 2 = ~~16~~ 8

At lowest rate of 30 - $\underline{\tau} =$

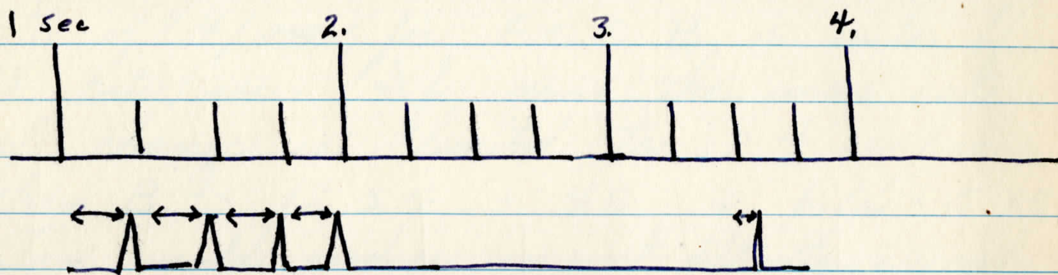
$2.00 - .24 = 1.76$ $B = 1.76/.02 = 88$

& bits = 8

In practical cases the sampling rate is const.

A second general scheme is the assignment of a value in time to the QRS complex. If another accurately known time scale is present this could be an economical scheme.

Assume Time is Known

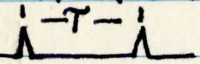


Worst case is above at rapid rate & assume 4 samples sec. for max rate : $\frac{25}{.02} \cong 12$ – this is 5± bits X $4^5/_{600}$ = ~~16~~ 20 bits.

Another general scheme is to transmit only changes in rate. For safety some redundant, repetitive signal to check the basic deviation point would be required. The maximum rate of change is the crucial factor here for although the average bandwidth may be small the overall bandwidth to accomodate these changes must be retained. This may be demonstrated by using the previous ~~same~~ values.

For a rate of 250 ~~bit~~ B/S = 12 X 4 = 48 + bits/sec = 7 & c̄ rate of 30; BS = 50 & total B ~~bits~~, for 2 sec. is 100 - adequate syllable ~~word~~ length must be provided for this and at the max. sample rate of 4/~~sec~~s for 800 B + rate of 9 bits/sec. If now the max. rate of change is limited to .24 sec/sec then B = 12 X 4 = 48 + bits = 5 X4 = 20 This would produce no great difficulty except that arrythmias c̄ rate of change greater than this would not be detected or at least recorded in their true value.

Recap of R-R transmission: time interval of R-R transmission (T) $\bar{c}$ an accuracy



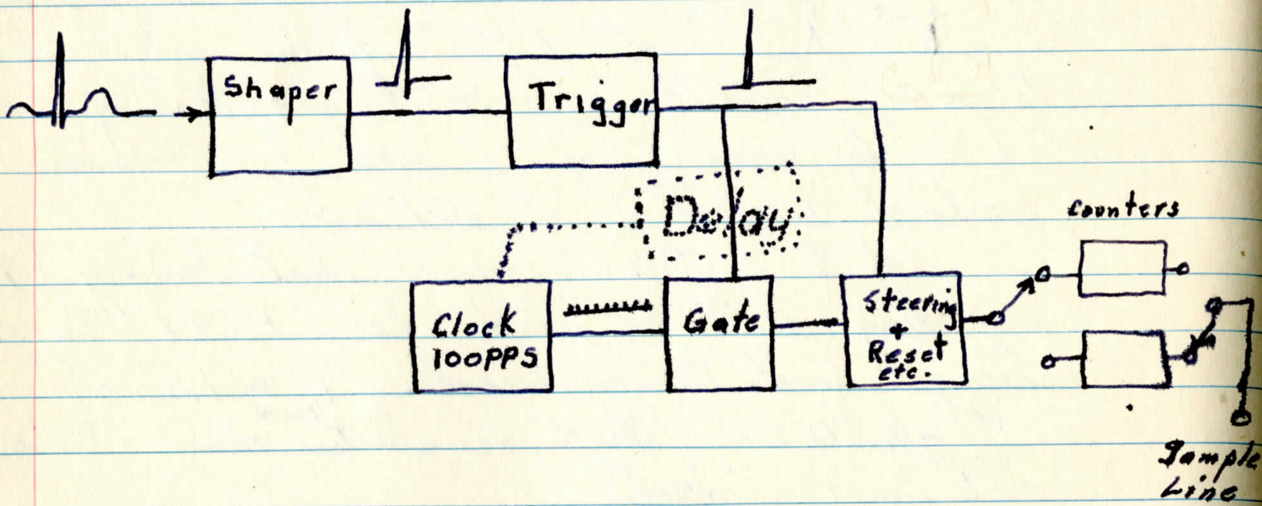
"of 1%" from rates of 40 to 200 beats/min. Will arbitrarily set "1%" to mean of 1 sec. for minimum <u>unit of resolution</u> of <u>.01 sec</u>.

Will further assume that binary (2^n) encoding will be used for PCM tape storage and transmission. $T_{40BPM} = 1.5$ sec. $T_{200BPM} = 0.30$ sec. Arrythmias shorter than .3 sec. will not be considered.

A. The most elementary method of encoding and measurement & transmission is to convert each QRS complex into a short standard pulse and count the .01 sec. intervals in each R.R. interval. A single sample rate of 4 samples/sec. is more than adequate to cover the max. rate. This will give:

at 40 BPM 1.5 sec. or 150 units for 9 binary bits/sample & at 4 samples/sec = 36 bits/sec. At the maximum heart rate of 200 the number of units/sample is 30 for a bit number of 6 and $\bar{c}$ 4 samples/sec has a rate of 24/sec.

Block diagram & waveforms of possible arrangement of A.



B. as above except dotted unit is added.

B. Another possible scheme which will effect some saving will be the insertion of a dead space into each sample period of the pulse. This can be as long as τ corresponding to the highest rate or .3 sec. which amounts to a saving of 150 - 30 = 120 units in the worst case of lowest pulse rate c̄ a sample bit number of 8 & max. rate of 32 bits sec. No loss of data would accrue from this method and its incorporation would be simple. (see opposite page).

C. Another basic variation is the transmission of only the difference from one pulse to the next. The number of binary code bits here will be determined by the maximum rate of change from sample to sample. For example if chose to limit the rate of change of interval to .3 sec. /sample - the max. bandwidth then becomes 30 units sample or 6 binary bits / sample + at 4 samples / sec.; 24 bits second. This places a number of limitations on the data transmitted. Using the value chosen at 200 beats / sec. a change of ≥ ±100 Beats / sec / sample may be transmitted but at low pulse rates, say of 50, only -10 + +22 Beats / sec / sample change can be accomodated. The most severe limitation is placed on accurate timing of extra-systoles. The rate of time change would again be limited to .3 sec. and at low rates this would be much less than the required change. See opposite page. In addition a steering pulse must be added to the code to give direction of change. This now raises the sample bits to 7 and bit rate to 28 sec. In addition some method of checking and or correcting in terms of absolute R-R interval would be mandatory. This would ~~in all probability~~ add to either the sample number or bit/sample such that the question of any real gain is questionable unless rates of change are severly restricted and there will be an additional cost of increased complexity.

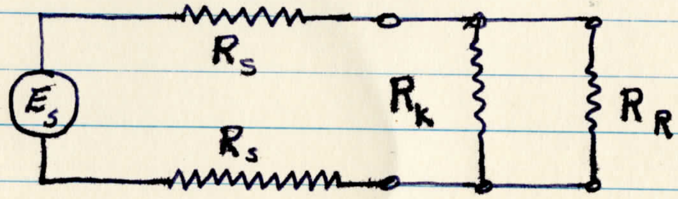
D. Some form of (real) time scale will be present c̄ a relatively high resolution. In addition the sample rate will be crystal controlled and the sample interval accurately know. Advantage may be taken of this to drastically reduce the transmitted information as follows - Using the same sample rate of 4 samples/sec let us assume each sample ~~point~~ is precisely known in real time to ≥ .01 sec. the min. resolution period. The time relation of the prev. R pulse to the nearest sample may be measured. At a rate of 4 samples sec. this will be a max. of 25 units or or 6 bits of code and a max. rate of 24 bits/sec. In this case this is the worst case since a lower heart rate will not exceed the sample bit number & the sample will be reduced.

Even if the individual sample time is ~~not or~~ unknown the sample period will be accurately known and these may be summed, if required, to give an equally accurate R-R interval.

Electrodes: Prob. - to determine optimum configuration for electrode to obtain lowest signal impeadance, lowest noise (artefact), simplest application, longest life of application, minimum of irritation and contamination of atmosphere.

As a prelimary our early epoxy/silver units were c̄ bilat. tape will be tried for life of application, and impeadance.

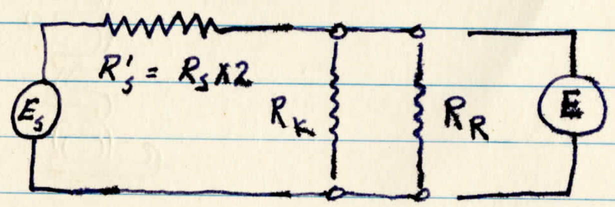
I mpeadance will be measured by shunting a known impeadance across the electrodes when connected to a high impeadance recorder and by measuring the change in signal voltage.



R_R = input imp. of recorder $> 10^6\,\Omega$

R_K = res. of Known subs.

R_s = imp. of souce from skin electrode, etc.



$$\frac{E_s}{\frac{R_K R_R}{R_K + R_R} + R_s'} = \frac{E}{\frac{R_K R_R}{R_K + R_R}}$$

R_K is on the order of $R_R/10^3$ and R_R is neglegible.

a

$$\frac{E_s}{R_s' + R_K} = \frac{E}{R_K}$$

With $R_K = \infty$ $E = E_s$

& With $R_K = R_s'$

$$\frac{E_s}{2R_K} = \frac{E}{R_K} \qquad E = \frac{E_s}{2}$$