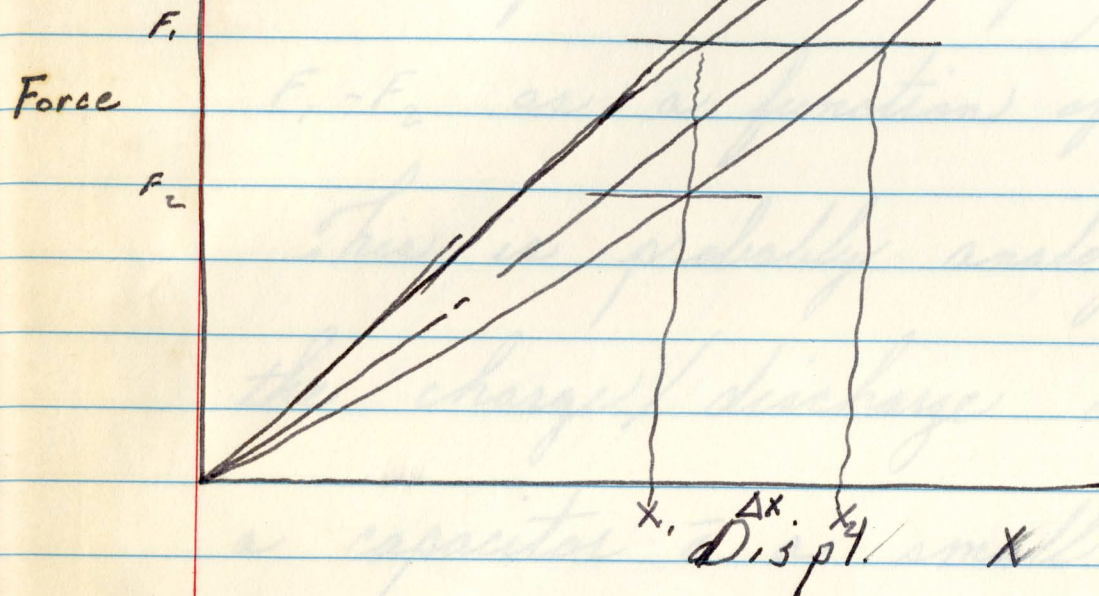


Hysteresis

7530-286-6951
FEDERAL SUPPLY SERVICE

16-74548-2 GPO

$$\text{Rate } F = kX$$



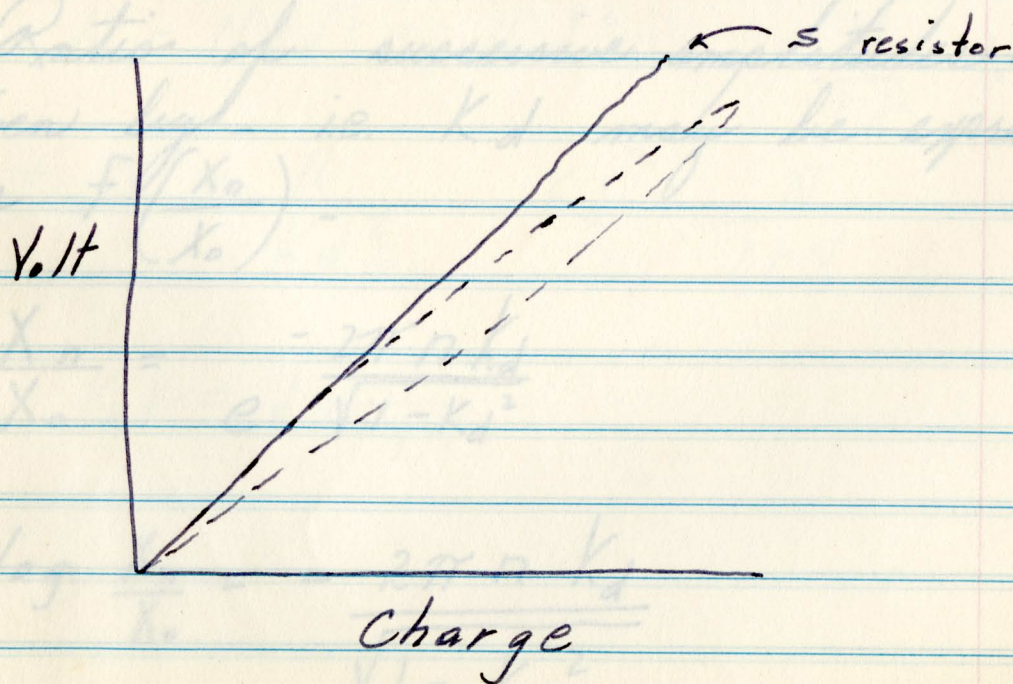
$$\text{Hysteresis} = \frac{\Delta x}{x}$$

Could find little or nothing in
the literature or in discussion c. Mr.
Gebow of Chatillon on the
shape of the above curve.

What is wanted is a
measure of the loss of force

$F_1 - F_2$ as a function of $\frac{dx}{dt}$.

This is probably analogous to
the charge/discharge curve of
a capacitor & a small resistance
in series.



Rather than to attempt the solution from static curves which could at best be measured with difficulty, a dynamic experiment will be attempted.

$$K_d = \frac{R}{2\sqrt{KM}} \quad + \quad R = K_d 2\sqrt{KM}$$

Ratio of successive amplitudes is given by - i.e. K_d may be expressed as $f\left(\frac{X_n}{X_0}\right)$ -

$$\frac{X_n}{X_0} = e^{-\frac{2\pi n K_d}{\sqrt{1-K_d^2}}}$$

$$\log \frac{X_n}{X_0} = -\frac{2\pi n K_d}{\sqrt{1-K_d^2}}$$

$$\alpha \left(\log \frac{X_0}{X_n} \right) = \frac{2\pi n K_d}{\sqrt{1-K_d^2}}$$

$$M^2 = \frac{4\pi^2 n^2 K_d^2}{1-K_d^2}$$

$$N = \frac{X_0}{X_n} = \frac{1}{1-K_d^2}$$

$$\left(\frac{M^2}{4\pi^2 n^2} \right) = \frac{K_d^2}{1-K_d^2} = \frac{\left(\log \frac{X_0}{X_n} \right)^2}{4\pi^2 n^2}$$

$$N = \frac{K_d^2}{1-K_d^2}$$

$$N - NK_d^2 = K_d^2 \quad 1 = \frac{N}{K_d^2} - N$$

$$1 + N = \frac{N}{K_d^2} \quad K_d^2 = \frac{N}{1+N}$$

$$K_d = \sqrt{\frac{N}{1+N}}$$

$$K_d = \sqrt{\frac{\left(\log \frac{x_0}{x_n}\right)^2}{4\pi^2 n^2} \left/ 1 + \frac{\left(\log \frac{x_0}{x_n}\right)^2}{4\pi^2 n^2} \right.}$$

By taking half amplitudes
 $x_n = \frac{x_0}{2}$

$$\log 2 = 0.3 \quad (0.3)^2 = 0.09$$

Take as example $n = 200$

$$K_{d_{200}} = \sqrt{\left(\frac{9 \times 10^{-2}}{12.56 \cdot 4 \times 10^4} \right) \left/ 1 + \frac{9 \times 10^{-2}}{50.24 \times 10^4} = .18 \times 10^{-6} \right.}$$

$$\approx \sqrt{18.} \times \sqrt{10^{-8}}$$

$$\approx 4 \times 10^{-4}$$

A simpler approximation
for $K_d < 0.1$ is

$$\frac{X_1 - X_2}{X_1} \approx 2\pi K_d \text{ —}$$