Saturday - April 8

Dear Bill

Here are

(a) The draft in form for you to complete

ie - add changes to section on stability
- check & insert needed corrections
- get drawings made
- type & send on way.

(b) 1 copy of what we sent to RSI last April to get out a recent paper

(c) 1 copy of what was finally produced

This will give your girls an idea of what they accept.

(We had to redo our original draft making certain that everything was in the CGS system!)

Regards
Ev

Gad - am I glad to get this out of my hair!

April 8th

Dear Bill

After my letter of last Sunday (enclosed herewith) several other things broke loose and I have done a hell of a lot of revising. No kidding – I have done a lot of work on this since I saw you (it would, of course, have been much more efficient if we could have worked together!) I am pretty well satisfied with what we now have.

Now it needs

(a) your comments on stability

(b) your revision of my added section on pressure dependence (if I have some things wrong.)

(c) drawings

(d) overall check by you

(e) final typing

TO AID IN LATTER, I SENT (ALONG WITH THE DRAFT WHICH WENT AIRMAIL & SPECIAL DELIVERY) A COPY OF OUR PAPER AS IT WENT TO R.S.I AND A COPY OF HOW IT CAME OUT.

Which reminds me – I could not find reference 1 – This was an article on plate fulcra which you sent to me, I know I have it, I checked something in it during the last 2 weeks – but now I can't find it! Ref #2 is the one by Yu, Yee-Tak as mentioned in Baker & Woolams paper from SRI on the Dynamics of a Mass Measuring Device.

Regards
Ev.

Didn't send till 8th → Sunday April 2

Found out later I wanted to change many things - took another 6 days to get paper on its way! for April 8th

Dear Bill

I have (in addition to completing the paper) put in quite a bit of extra time on examining the various fits which Lune got - these have worried me & ~~I~~ felt that we had better know what was going on before we let the paper go out. I am now satisfied with our results as they stand - so I will summarize what I found out and you can file it away - I do not think these notes need to be ~~raff~~ referred to in this paper though we might want to use them on the big effort.

*** Have changed my mind after writing this letter & have made a slight revision to include these results

On our latest data, the computer sheet which you sent me had ~~three~~ two solutions

Linear - $T^2 = 4.229000 + 0.00872874m$

Quadratic - $T^2 = 4.229885 + 0.00870838m + 0.0000000449816\,m^2$

(Forget the cubic and quartic for the moment!)

Question 1 Now how do these compare with the most probable value out of each of those 6 sets of 50 data points which you & I ran off and which he ran a least squares fit to get the above results?

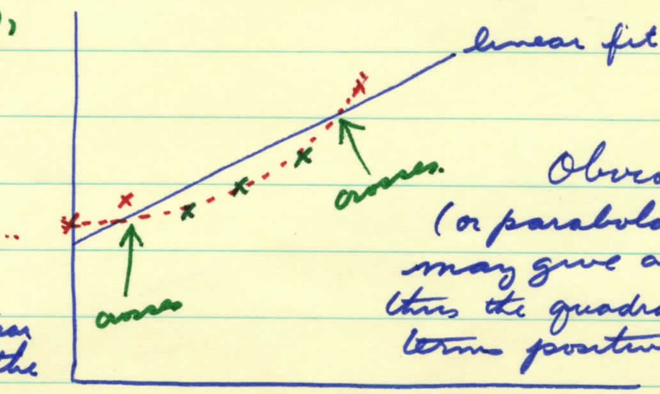
Mass	Observed T (peak)	Linear fit		Quadratic fit		Cubic fit	
		Predicted T	% Error	Predicted T	% Error	Predicted T	% Error
0	2.05667	2.05645	− 0.010	2.05667	-0.0000_8	2.05665	-0.0009_1
15.4992	2.08925	2.08909	− 0.008	2.08923	− 0.001	2.08922	-0.001_3
57.0778	2.17408	2.17422	+ 0.006	2.17419	+ 0.005	2.17420	$+0.005_7$
112.8566	2.28337	2.28344	+ 0.003	2.28326	− 0.005	2.28327	-0.004_2
226.5438	2.49095	2.49127	+ 0.013	2.49099	+ 0.001	2.49097	$+0.000_8$
453.9059	2.86216	2.86200	− 0.006	2.86216	0.000	2.86216	+ 0.000

Now lets examine this set of figures:

1st - irrelevant to our analysis but of interest - is that Mr Lurie has his sign wrong on his % errors. If the number you predict is larger than the measured value your error is a positive one - in which case your correction (to be applied to your predicted value) is a negative number. However we are interested in changes of signs of errors so it doesn't matter.

2nd Note under linear fit - how the errors go. This means our results look like

Note - quadratic starts above linear expression (see m=0), has a smaller slope (see 2nd coeff) but the m^2 term being + enables it to recover. Quadratic crosses the linear expression twice.



One may well be suspicious of a fit (here the linear case) which has the signs of the errors go - - + + + -

Obviously a quadratic (or parabola opening upwards) may give a better fit - thus the quadratic should have all terms positive and it does.

3rd This quadratic now has errors which jump around (in sign), which is more reasonable and there is a considerable decrease in the % error column - a real gain. It looks to me that we are quite justified in saying that this is a real gain in ~~fitting~~ the curve. Also ~~it looks as though~~ the loading is causing the period to be increased which is what I would expect in a normal non linear system.

Also - I presume this improvement has shown up consistent & similar on every run.

My Conclusion - The quadratic gives a much better fit. - even though the linear fit is good enough for our purpose

Question 2 Is the cubic a better fit than the quadratic?

Only very slightly - look at page 1 - the gain is so small that it is not worth the effort to use it and the gain doesn't mean much statistically or experimentally.

In fact I suspect that the reason that we get a better fit is that we are using one more parameter. In fact if we ~~had~~ went out to 6 terms in the series (5th power!) - since we started with 6 points we would have 6 equations in 6 unknowns & could get an exact fit!

My conclusions are these

(a) To the order of accuracy which was the original goal (0.1%), the linear fit is perfect. - In fact the linear fit is good to 1 order of magnitude beyond this. However the accuracy of the apparatus is even better than this - and it appears that a quadratic fit ~~is required~~ will bring even better agreement with the data. (out to at least 0.005% or better for all points)

(b) I do not think that the cubic fit is any essential gain over the ~~good~~ quadratic fit.

(c) I think that the quadratic fit would be explainable in terms of known phenomena (known to us that is) though I would not recommend that we get involved in this explanation! It would be interesting but the amount of effort & time involved could be exhorbitant.

2) constraints of experimental conditions

3) instrumental details

1 – Theoretical

The solutions of the general differential equations of an oscillating system ~~are~~ have been solved in detail since the same equations describes a number of physical systems including mechanical, electrical and acoustic.

The idealized system consists of a mass M, attached to a restoring force $F = KX$, provided by a massless spring attached to a rigid support. Motion of the particle is assumed to create a linear resisting force of $R\frac{dx}{dt}$.

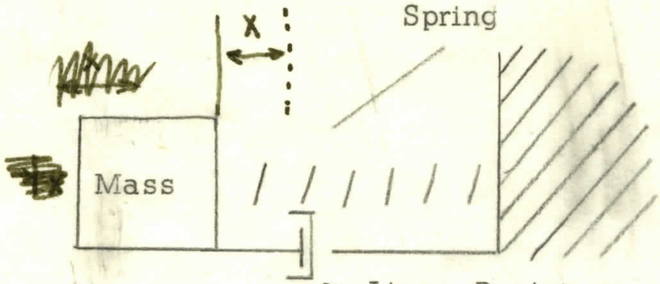


Fig. 1

The general equation of motion of such a system undergoing "natural" oscillations (ie, displaced from its position of rest and allowed to return with outside influence) in a single plane is:

1.1 $$M\frac{d^2x}{dt^2} + R\frac{dx}{dt} + KX = 0$$

There are two possible forms to the solution of the equation depending upon the relative amount of resistance present. If the resistance, R, is equal or greater than $2\sqrt{KM}$ then the mass will return to the position of equilibrium in an exponentional fashion. If, ~~however~~, as the case will be here, $\frac{R}{2\sqrt{KM}} < 1$, an oscillation about the equilibrium point will result. If the resistance is zero, an undamped or continuous oscillation will occur whose frequency is given by:

$$F_n = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{KG}{W}} \qquad \text{Eq. 1.1}$$

Where: F_n = undamped natural frequency (C.P.S.) (cycles per sec.) ~~where~~ with $R = 0$

K = spring constant (pounds/inches)

M = mass $\left(\frac{\text{pounds second}^2}{\text{inches}}\right) = \frac{W \text{ (pounds)}}{G \text{ (Inches/second}^2)}$

W = weight (pounds)

G (Austin, Texas) 979.283 cm/sec^2

= 385.54372 in/sec^2

1 incn (US) = 2.540005 cm.

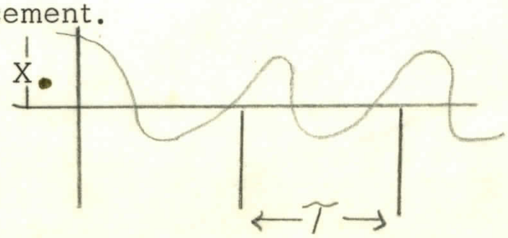
~~And $F_n = \frac{1}{T_n}$~~ ~~And whose period~~

The undamped natural period ~~$T = \frac{1}{F_n}$~~ T_n = time (secs.) = $\frac{1}{F_n}$

~~1.2 $T_n = \frac{1}{2\pi}\sqrt{\frac{W}{KG}}$~~

$$T_n = \frac{1}{F_n} = \frac{1}{2\pi}\sqrt{\frac{W}{KG}} \qquad \text{Eq. 1.2}$$

~~and whose~~ Amplitude will be equal to the amplitude of the original displacement.



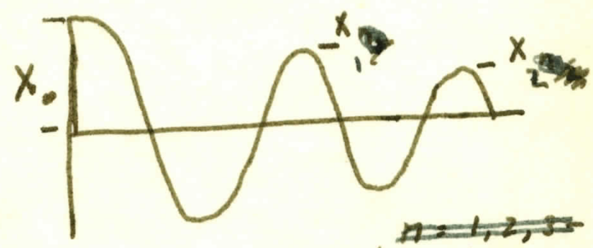


Fig. 1.2 A Undamped natural oscillation $R = 0$ B. Damped natural osc. $0 > R < 2\sqrt{KM}$

It the resistance is not zero, ie, energy is dissipated as in any practical system, both the frequency and amplitude will be modified from the undamped case.

The new frequency will ~~be shifted~~ be given by: ~~$W = 2\pi f$~~

$$\text{~~1.3~~}\quad F_d \cancel{\text{(damped)}} = F_n \cancel{\text{undamped}} \sqrt{1 - \left[\frac{R}{2\sqrt{KM}}\right]^2} \qquad \text{Eq 1.3}$$

F_d = damped natural freq. where $R \neq 0$ (Cycles per sec.)

In addition, the peak amplitude will decrease by Exponential

$$\frac{X_n}{X_o} = e^{-2\pi n \frac{R}{2\sqrt{KM}} \Big/ \sqrt{1 - \left(\frac{R}{2\sqrt{KM}}\right)^2}}$$

X_o = Amplitude of original displacement

X_n = peak amplitude of successive damped oscillations

n = number of cycles = 1, 2, 3 ...

The relationship of three successive peak amplitudes to the "damping ratio" is shown in Fig. 3.

Pg. 2-5 Vns HB

Damping Ratio, $K_d = \frac{R}{2\sqrt{KM}}$

Fig. 3

The two fundamen al questions that arise from purely theoretical considerations then are concerned with the time (frequency) determinations and knowledge of R, K&M to determine deviations from the natural frequency. The minimum time resolution ΔT required for idealized measurement of a mass change of ~~.1%~~ .1 pound is calculated here for a typical case of a 150 pound object at a period of approximately 1.2 seconds.

τ_1 - period (seconds) at 150 pounds

τ_2 - period (seconds) at 150.1 pounds

$\Delta T = \tau_2 - \tau_1$

W_1 = 150 pounds

W_2 = 150.1 pounds

K = 10#/in.

~~g (value for Austin, Texas) = 385.54372 inches/seconds~~^{2}

$$\tau = 2\pi\sqrt{\frac{W}{K\,G}}$$ Eq. 1.2

$T_2 = 2 \times 3.\cancel{4166}^{\,1416} \times .1\ 97311 = 1.2397444$ seconds

$T_1 = 2 \times 3.1416 \times .1\ 97245 = 1.2393297$ seconds

$\Delta T = .4147 \cancel{m} \times 10^{-3}$ seconds $\approx .5$ m Sec.

This order of time resolution may be readily obtained by a counter of 10^{-4} seconds ~~, counter~~ resolution (10^{-6} seconds resolution is routine) so measurement of time per se is not difficult, As will be shown later, the difficulty will arise from the distance resolution required to obtain this time. but It is obvious ~~however~~ that a stop watch will not suffice.

The next consideration, effects of resistance, cannot be determined with such accuracy. The resistance in the real case will not be lumped but will consist of several components ~~most of~~ which do not readily lend themselves to precise calculation. The two chief sources of resistance arise from: 1) the motion of the mass (man) through a viscous medium (air). 2) the dissipation of the spring. While the spring resistance will remain a system constant, the viscous resistance (air drag) will vary with subject configuration and surrounding atmosphere and cause variable deviations from the undamped natural frequency. Since the body will be irregular and variable and the atmosphere is presently unknown, only an estimation may be made of this effect.

Air "drag" at ~~aircraft~~ appreciable velocities may be calculated from $D = C_d \, S \, P$

P = dynamic pressure = $\dfrac{\rho V^2}{2}$

D = drag (pounds)

C_d = Coefficient of drag

= 1.28 for flat plate

S = Area (Ft.^{2})

insert

* P - 6

The aero - med lab uses values of CdS of ~~5 - 10 for~~ 5 ft.^{2} min to 10 ft.^{2} max. for the clothed human body. Taking the 10 ft.^{2} value will result in a R of $\approx$ 1.5 x 10^{-2} $\frac{\text{pounds} \cdot \text{second}}{\text{Feet}}$.

Using this value, very much a 'worst case,' in Eq. 1.3 for a nominal ~~u~~ undamped

Tod value of 1.24 0000 secs. which corresponds to approx. a 150# ~~w~~ man $\bar{c}$ K = " (?-ck) gives -

$$F_d = F_n \left(1 - \left(\frac{R}{2\sqrt{KM}}\right)^2\right)$$

ρ = density = 2.38×10^{-3} slugs/ft^3 for normal atmosphere

V = velocity (feet/second)

Resistance $R = \frac{\text{Force}}{\text{Velocity}}$ and combining this with the drag ~~force~~ equation:

$$R = \frac{C_d S \rho V^2}{2V} = \frac{C_d S \rho V}{2} \qquad \text{Eq. 1.3}$$

Assuming a flat plate area of 3 feet^2 for a seated man in a normal atmosphere with a velocity of 1 foot/second (this is greater than our maximum), the drag is calculated to be

$$R = \frac{1.23 \times 3 \times 2.38 \times 10^{-3} \times 1}{2} = 4.57 \times 10^{-3} \text{ pounds/feet/seconds}$$

* We may next note the maximum allowable effects of resistance for the accuracies desired (auxillary calibration is possible but undesirable)

We have seen that a tolerance of ~ .5 $\times 10^{-3}$ seconds in time is allowable ~~in time for an accuracy of ±.1% in mass determination~~ a change of .1 pound at 150 pounds and for convenience these times figures will be used here ~~to calculate max allowable resistance~~. Eq - 1.3 allows calcualtion of the effect of R and rearranging this in terms of period ~~and~~ Eq 1.4 will allow one to calculate the maximum tolerable R for the given ΔT ~~damping ratio~~: $K_d = \frac{R}{2\sqrt{KM}}$ damping coefficient

assuming there are no spring losses:

$$K_d = \sqrt{1 - \left(\frac{T_n}{T_d}\right)^2} \qquad \text{Eq. 1.4}$$

R = Mechanic resistance (pounds/inches/seconds)

K = Spring constant = 10 pounds/inches

$M = \frac{W}{g}$ = Mass = $\frac{150 \text{ pounds}}{g}$

$\approx$ .389 pounds second^2/inches

T_n = Undamped period (seconds) ~~i.e. ideal case~~

= 1.2393297

T_d = damped period (seconds) = 1.2397444

$$K_d = \sqrt{1 - \left(\frac{1.2393297}{1.2397444}\right)^2} = \sqrt{1 - .99933} = 2.58 \times 10^{-2}$$

2.1 — Some of the conditions imposed or implied by weightlessness include:

2.1.6 ~~limit~~ severe limitations as regard to weight, size, power, space

2.1.2 ~~limited skill of operators~~ minimum possible skill levels and total time of operation ~~implicit~~

2.1.3 a variety of environmental conditions including high vibration and G-forces

2.1.4 reliability and ~~simplicity and~~ as a corrallary of this simplicity

2.1.5 limited mass of space ship

2.1.01 physical characteristics of the human body